GeoScience Engineering

FLOOD FREQUENCY ANALYSIS IN THE CATCHMENT OF OUED MAZAFRAN IN THE NORTHERN OF ALGERIA

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ABSTRACT

Extreme flood events are commonly described by several dependent characteristics, such as duration, volume and peak flow. In Algeria and North Africa, flood frequency analysis is conducted as a univariate approach focusing separately on each single of flood characteristics. The analysis of flood in oued Mazafran catchment by the flood frequency analysis is crucial for understanding and managing flood risk, designing infrastructure and planning for floodplain management. To perform flood frequency analysis, the series of the annual maximum flow discharge of Fer a Cheval station is fitted to a probability distribution. Well, several probability distributions can be used to fit this series such as Gumbel, Log Normal, Log Pearson type III, and more. The Choosing a single distribution probability has become an important question in hydrology frequency analysis. Therefore, the use of the decision support system that is based on the heavy tails of distribution probability analysis conducts to the use of sub-exponential distributions and information Criterion to the Gumbel distribution for this analysis.

Keywords: Catchment; Distribution; DSS; Flood; Frequency analysis; Series.

1 INTRUDUCTION

Floods are caused by extreme rainfall and their evolution depends on geomorphological characteristics of the catchment and to the urban expansion and its consolidation around rivers. According to the change in climate, the change of flood regime is a major factor that leads to increased exposure to the risk of flooding [1]. In Algeria, floods are considered to be a major natural calamity. The flooding of November 2001 in Algiers, the floods of oued Bechar and oued M'Zab in Ghardaïa October 2008 as examples are an adverse impact on the people and on infrastructure. The phenomena of floods manifest themselves in a catastrophic manner thus constituting a major constraint for economic and social development. Floods are the most destructive and even the most frequent natural disasters and cause significant human and material damage. The oued Mazafran in the north of Algeria is often confronted with floods which can be caused flooding in the surrounding region, the flood of March 1974 caused 52 deaths, destroyed 4570 houses and 13 bridges [2].

The occurrence of extreme precipitation leads to increase in the magnitude and frequency of extreme floods [3]. Estimating of flood can be carried out using methods depending on data resources and time availability [4]. So, the frequency analysis discharge of annual maximum instantaneous or annual maximum peak flow data (Q_{IX}) may be used for estimation of the discharge Q_{IXT} for return periods T, which is essential for any studies of the hazard and management relation of flooding. Relating the magnitude of extreme events to their frequency of occurrence, through the use of probability distributions, is the principal aim of the frequency analysis [5].

Flood frequency analyses involves computing statistical information of a given annual maximum instantaneous discharge. Frequency distribution is generated gives the likelihood of various discharges as a function of return period. Many distributions may be used for flood frequency analysis. The Gumbel's distribution, the log-normal distribution and log-Pearson Type III distribution are standard flood frequency distributions used by US Federal agencies such as Federal Emergency Management (FEMA) and US Geological Survey (USGS) can be used to

Vol. 70 (2024), No. 2
pp. 17–29, ISSN 1802-5420
DOI 10.35180/gse-2024-0109

estimate design floods with their own advantages and disadvantages [6]. But in hydrology of extremes, several distributions may be used for fitting annual peak flow series [7]. Therefore, a method focusing on the tail of the distribution should be preferred [8], although by the use of the Decision Support System (DSS) [9], which defines the class of distributions prior to a model selection practice with respect to tail behaviour of the time series of data [10]. The DSS conducts finally to the choice of the appropriate distributions for the frequency analysis.

2 STUDY AREA AND DATA SETS

The study has been conducted on the catchment of oued Mazafran (Figure 1) situated in the North of Algeria about 50 km from the west of Algiers between 2°15'and 3°00' of East longitude and 36°15' and 36°45' of North latitude. The area of the catchment is 1893 km^2 and its perimeter is 315 km, about 60 % of the catchment is mountainous. The catchment is elongated with a 131 km of length and 14.5 km of width. The stream of oued Mazafran is the junction of three main streams, oued Bou Roumi, oued Djer and oued Chiffa, where the risk of flooding is significant, which needs to be monitored during rainfall as it has become stronger and more intense in recent years [2, 11].

The catchment of oued Mazafran (Figure 1) is characterised by a Mediterranean climate, semi-arid to arid, characterized by dry and hot summers and cold and humid winters. The rainfall is abundant but irregular; the catchment receives annually between 600 mm and 900 mm of precipitation [2], causing significant flooding especially in the stream of oued Mazafran.

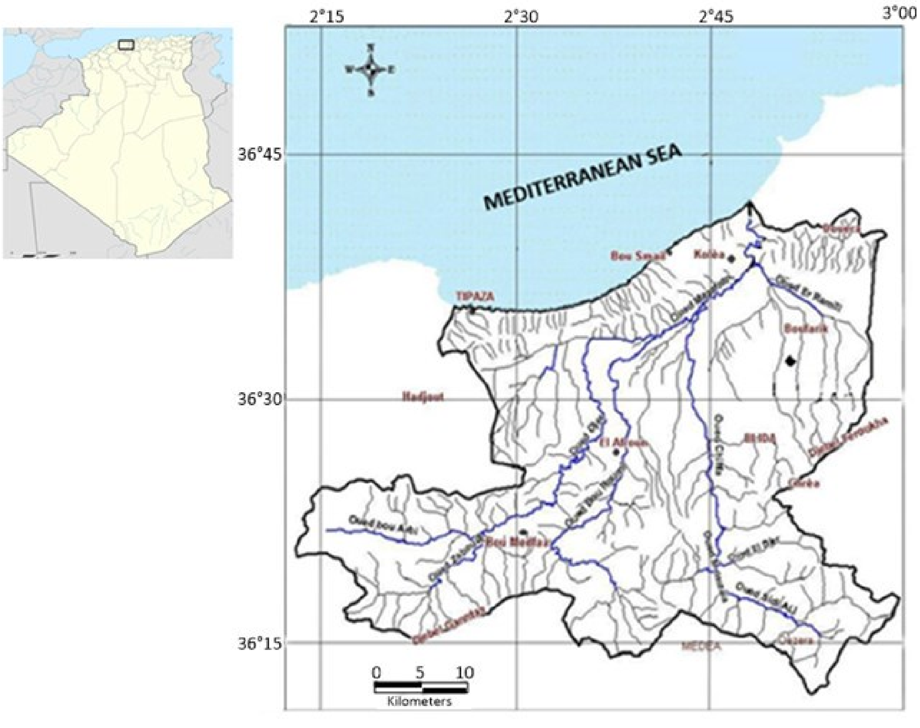

Figure 1. *The catchment of oued Mazafran*

The hydrometric data set used in this study were provided by the National Agency of Hydraulics Resources (ANRH) of Algiers, it consists of the time series of annual maximum instantaneous discharge (QIX) recorded at Fer à Cheval gauge station for the periods 1969/1970 to 1987/1988, 1990/1991 to 1994/1995 and 1997/1998 to 2011/2012 (Figure 2). The number of element in the time series is 42, with a mean of 269.25 m3/s and standard deviation of 179.62 m3/s, the min and max values are respectively 18 m3/s and 861 m3/s.

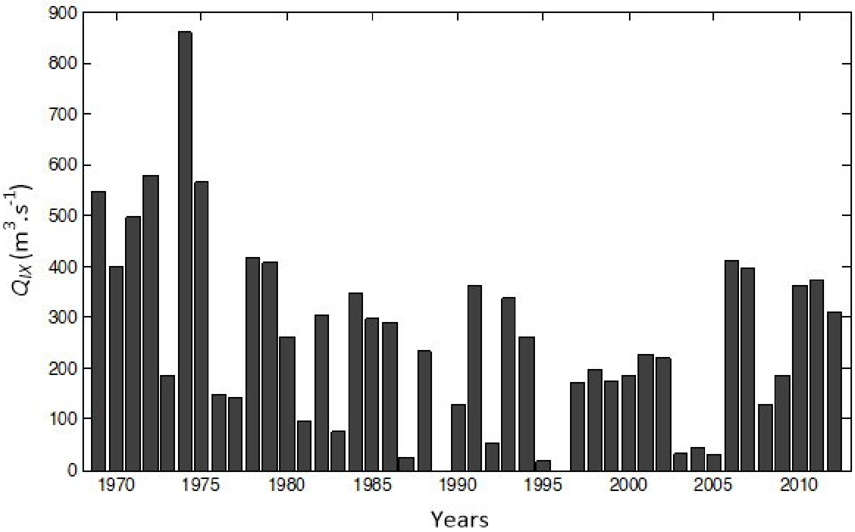

***Figure 2.** Annual instantaneous maximum discharge at Fer à Cheval gauge station*

By the use of Wald-Wolfowitz test [12], Mann-Whitney test [13], Mann-Kendall test [14, 15] and Grubbs test [16], the series of Q_{IX} is independent, homogeneous, stationery and with no outlying values (Table 1). So, it may be used for frequency analysis study.

***Table 1.** Sampling tests of the annual instantaneous maximum discharge series*

Type of the test	Calculated Value	Critical Value
Wald-Wolfowitz test for independence at 5%	1.860	1.960
Mann-Whitney test for homogeneity at 5%	1.698	1.960
Mann-Kendall test for stationarity at 5%	1.170	1.960
Grubbs test for outliers at 1%	3.294	3.404

3 MATERIALS AND METHODS

3.1 The use of decision support system

The estimation of the event Q_{IXT} requires the knowledge of the nature of the distribution of the population of the annual instantaneous maximum discharge Q_{IX}. So, a number of distributions probabilities may be used to fit a series of Q_{IX}. Thus, the use of the Decision Support System (DSS) [9, 17] may conduct to the choice of the class of distributions prior to a model selection practice with respect to tail behaviour of the series of data [10]. The DSS is based on the study of the tail behaviour of extreme event distributions by two classifications and four graphical criteria. The first graphic is the log–log plot, well the series of Q_{IX} is fitted to Zips or Pareto distribution [18], well its probability density function is given by:

$$P(X = Q_{IX}) = \frac{\alpha - 1}{Q_{\min}} \left(\frac{Q_{IX}}{Q_{\min}} \right)^{-\alpha} \qquad (1)$$

Where, α is the only parameter (tail) and $Q_{\min}$ is the minimum value existed in the population of Q_{IX}, α may be estimated by the maximum likelihood method [19, 20] as:

Vol. 70 (2024), No. 2
pp. 17–29, ISSN 1802-5420
DOI 10.35180/gse-2024-0109

$$\alpha = 1 + \frac{n}{\sum_{i=1}^{n} \ln \frac{Q_{IXi}}{Q_{\min}}} \quad (2)$$

Hence, $Q_{\min}$ is obtained as a function of α by:

$$Q_{\min} = \exp\left(-\frac{1}{\alpha - 1} + \frac{1}{n} \sum_{i=1}^{n} \ln Q_{IXi} \right) \quad (3)$$

To estimate the value of α [21, 22], let's considering the discrete probability density function of Pareto given by [23]:

$$P(X = k) = \frac{k^{-\alpha}}{\zeta(\alpha)} \quad (4)$$

Where, $\zeta(\alpha)$ is the Riemann Zeta function [25], which is given by:

$$\zeta(\alpha) = \sum_{m \geq 1} \frac{1}{m^{\alpha}} \quad (5)$$

The estimation of α by maximum likelihood method gives:

$$\frac{\zeta'(\alpha)}{\zeta(\alpha)} = -\frac{1}{n} \sum_{i=1}^{n} \ln Q_{IXi} = -5.29 \quad (6)$$

$\zeta(\alpha)$, is the derivative of the Riemann Zeta function. The value of the ratio $\zeta'(\alpha)/\zeta(\alpha)$ can be generated on most modern mathematical and engineering calculation programs [24]. So, $\alpha = 1.17,$ from the equation (3), $Q_{\min}$ is calculated and is equal to 0.55 $m^3.s^{-1}$. Thus, the series of Q_{IX} may be fitted to Zips or Pareto distribution (Figure 3).

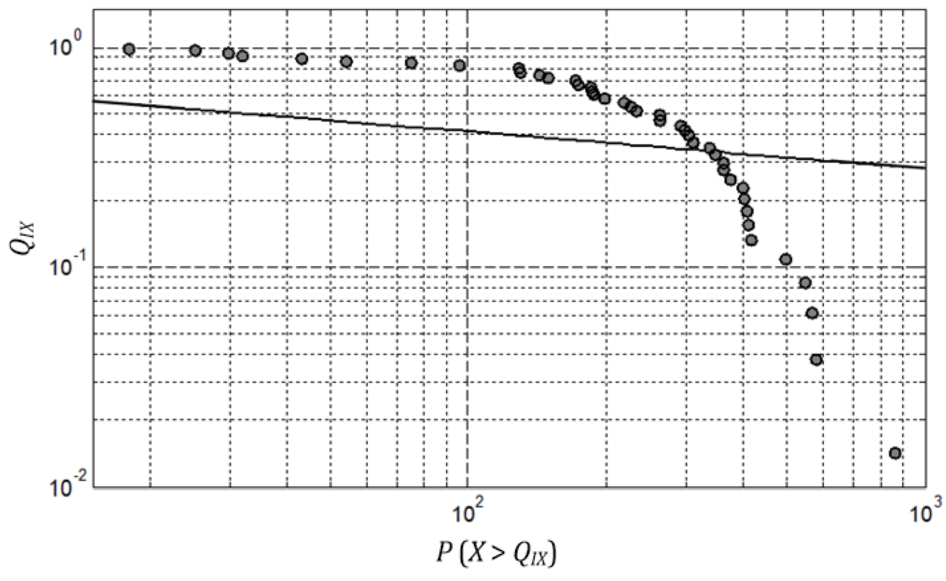

Figure 3. *Log-Log plot for the regularly varying distributions*

Vol. 70 (2024), No. 2
pp. 17–29, ISSN 1802-5420
DOI 10.35180/gse-2024-0109

According to the Log-Log plot (Figure 3) of the fitting the series of Q_{IX} to Pareto distribution, the DSS conduct to consider that the distributions for frequency analysis of series of Q_{IX} may be either exponential distribution (Class E) or sub-exponential distributions (Class D). To discriminate between these two classes of distributions it has to use the mean excess function (Figure 4) [26].

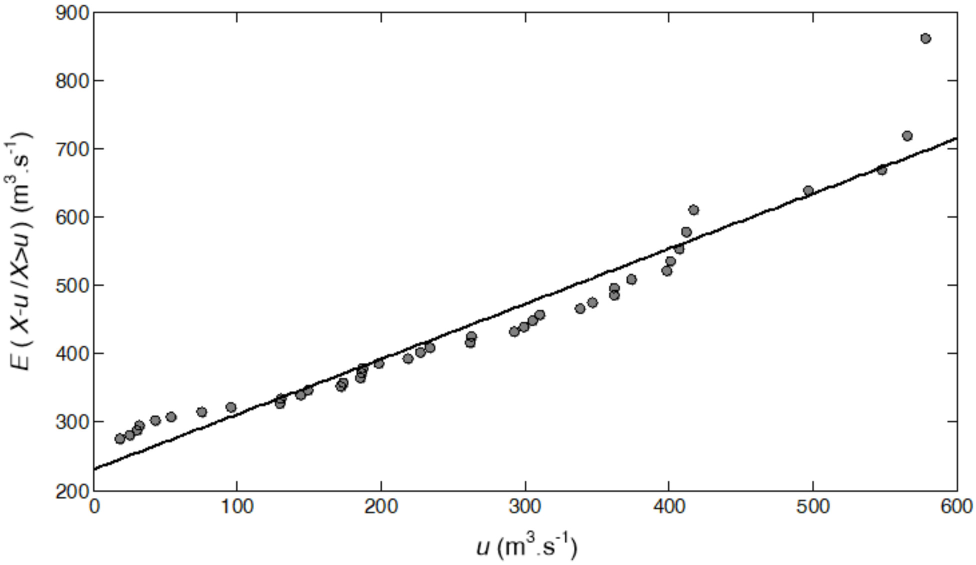

Figure 4. *Excess mean function plot*

The mean excess function has a positive slope. So, the sub-exponential distributions of the class D may be used for the analysis of the series of Q_{IX}. Thus, the confirmation of this decision may be done by the generalized Hill ratio plot (Figure 5) [27] or with the method based on statistic of Jackson (Figure 6) [28].

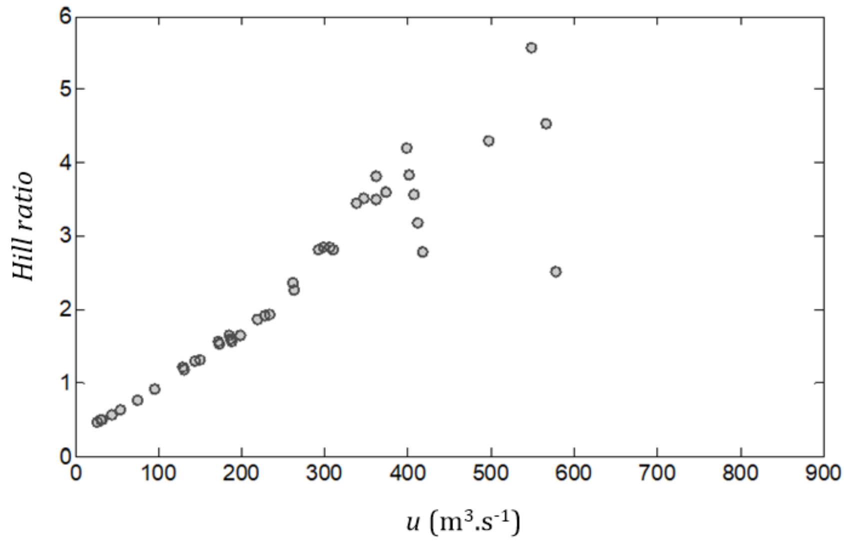

Figure 5. *Generalized Hill ration plot*

Vol. 70 (2024), No. 2
pp. 17–29, ISSN 1802-5420
DOI 10.35180/gse-2024-0109

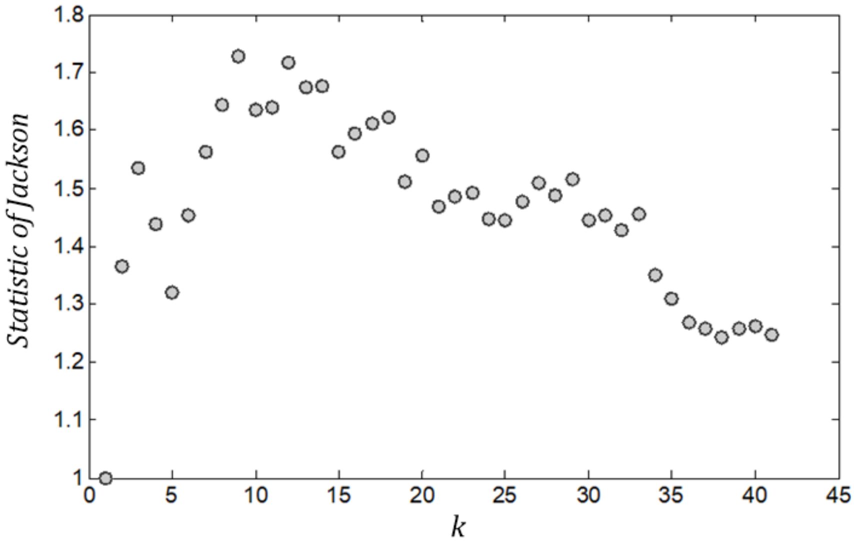

Figure 6. *Statistic of Jackson plot*

The Generalized Hill ratio plot (Figure 5) shows that there is a tendency to an oblique mean stability according to the majority of the points and the statistic of Jackson plot is irregular and is diverges from 2. Therefore, those plots [9] confirm the use of the sub-exponential distributions of the class D for the frequency analysis of the series of Q_{IX}.

3.2 Fitting of the series of QIX

The sub-exponential distributions applied in the frequency analysis in hydrology of extremes are Gumbel, Gamma, Pearson Type III and Halphen B distributions. Those distributions are used for the frequency analysis of the series of Q_{IX} of oued Mazafran.

3.2.1 Case of Gumbel distribution

The Gumbel distribution is given by its cumulative distribution function:

$$F\left(Q_{IX}\right)=\exp\left(-\exp\left(-\frac{Q_{IX}-m}{a}\right)\right) \quad (7)$$

m and a are the location and scale parameters. By the use of the probability weighted method (PWM) [29], the estimation of a and m are respectively equal to 143.90 $m^3.s^{-1}$ and 186.20 $m^3.s^{-1}$ (Figure 7).

3.2.2 Case of Gamma distribution

The Gamma distribution with the scale parameter a and the shape parameter k is given by the following probability density function:

$$f\left(Q_{IX}\right)=\frac{1}{a\Gamma\left(k\right)}\left(\frac{Q_{IX}}{a}\right)^{k-1}\exp\left(-\frac{Q_{IX}}{a}\right) \quad (8)$$

Vol. 70 (2024), No. 2
pp. 17–29, ISSN 1802-5420
DOI 10.35180/gse-2024-0109

Where, $\Gamma(.)$ is the Gamma function [30]. By the method of moments, a = 119.83 $m^3.s^{-1}$ and k = 2.25 (Figure 8).

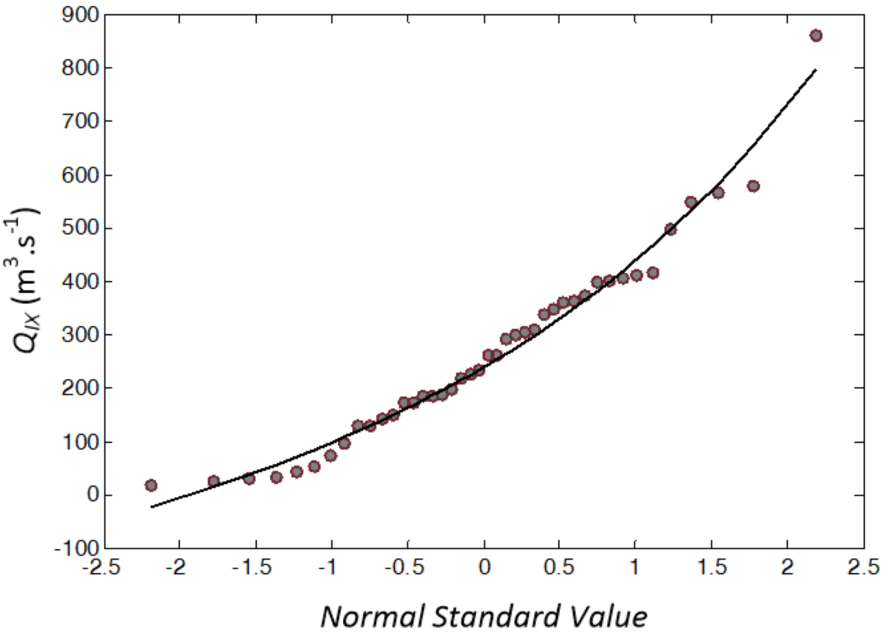

Figure 7. *Fitting to Gumbel distribution (PWM)*

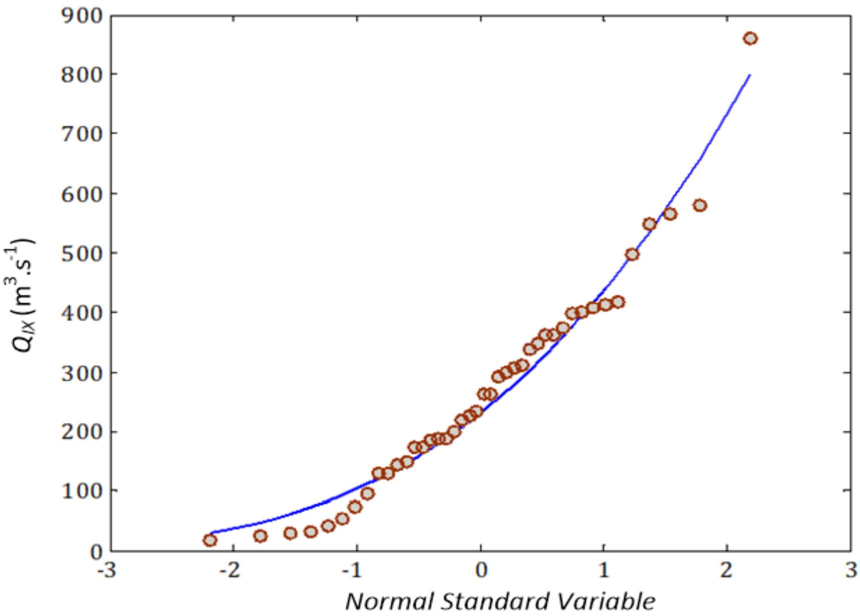

Figure 8. *Fitting to Gamma distribution (MM)*

3.2.3 Case of Pearson Type III distribution

The Pearson Type III distribution with location parameter m, scale parameter a and the shape parameter k, is given by its density probability function:

$$f\left(Q_{IX}\right)=\frac{1}{a\Gamma\left(k\right)}\left(\frac{Q_{IX}-m}{a}\right)^{k-1}\exp\left(-\frac{Q_{IX}-m}{a}\right) \quad \textbf{\textit{(9)}}$$

The method of moments (MM) gives, the estimate of *a*, *m* and *k* as -120.65 $m^3.s^{-1}$, 186.20 $m^3.s^{-1}$ and 4.71 (Figure 9).

3.2.4 Case of Halphen B distribution

The Halphen B distribution with scale parameter *a*, and two shape parameters *k* and *υ*. The probability density function of Halphen B is given by:

$$f\left(Q_{IX}\right)=\frac{2\,Q_{IX}^{2\upsilon-1}}{a^{2\upsilon}ef_{\upsilon}\left(k\right)}\exp\left(-\frac{Q_{IX}^{2}}{a^{2}}+k\,\frac{Q_{IX}}{a}\right) \quad \textbf{\textit{(10)}}$$

Where, $ef_{\upsilon}\left(.\right)$, is the Halphen's exponential factorial function [31]. The estimation of the parameters *a*, *k* and *υ* by the method of the maximum likelihood (MLE) are a = 373.43 $m^3.s^{-1}$, k = 0.44 and υ = 0.59 (Figure 10).

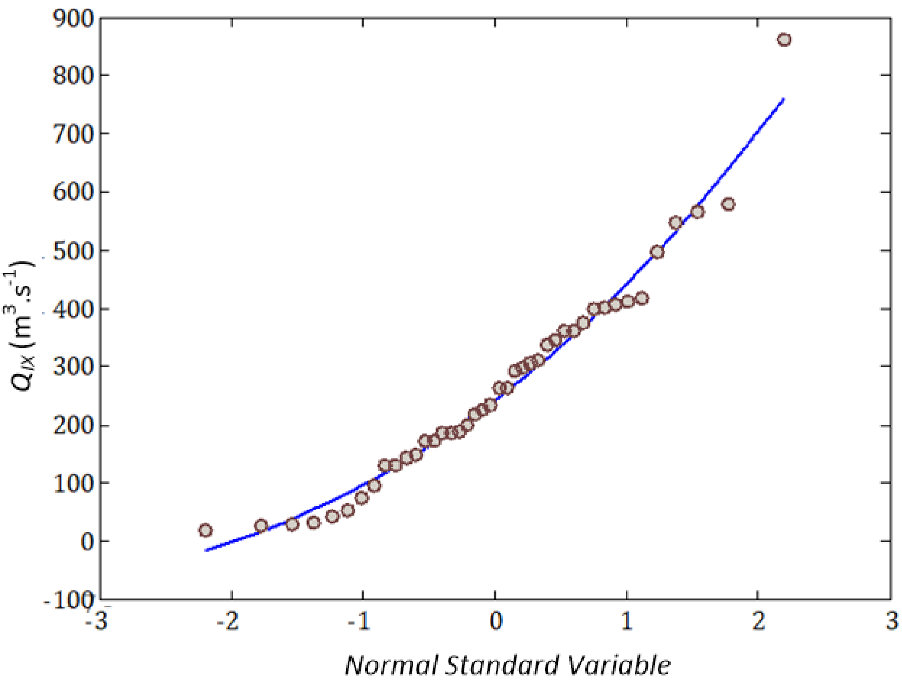

Figure 9*. Fitting to Pearson Type III (MM)*

Vol. 70 (2024), No. 2
pp. 17–29, ISSN 1802-5420
DOI 10.35180/gse-2024-0109

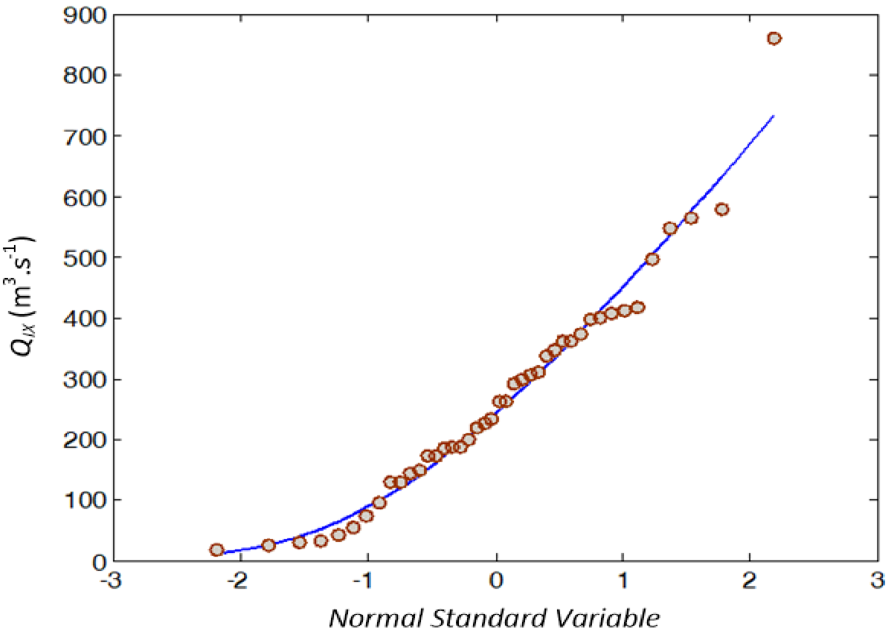

Figure 10. *Fitting to Halphen Type B (ML)*

4 RESULTS AND DISCUSSION

Goodness-of-fit test of the used distributions (Table 2) may be done by the probability plot correlation coefficient (*PPCC*) [32, 33], root mean square deviation (*RMSD*) [34] and the test of Kolmogorov–Smirnov (*KS*-test) [35]. The values of *PPCC* and *RMSD* assess the fitted distribution at a site by summarizing the deviations between observed discharges and computed discharges. The *KS*-test is a nonparametric test, the computation of D_{ob} as the maximum of the difference between cumulative observed frequencies F_i and the theoretical distribution $F\,(Q_{IX})$:

$$D_{ob} = \max\left|F_i - F\left(Q_{IXi}\right)\right| \quad \textbf{(11)}$$

D_{ob} must be lower than the critical values $D_{n,\alpha}$, where α the significance level and n is the sample size. When n is over 35, $D_{n,\alpha}$ is calculated for α = 1% and α = 5% by:

$$D_{n,\ 1\%} = \frac{1.63}{\sqrt{n}} = 0.252 \quad \textbf{(12)}$$

$$D_{n,\ 5\%} = \frac{1.36}{\sqrt{n}} = 0.210 \quad \textbf{(13)}$$

According to obtaining results (Table 2), all the observed Kolmogorov–Smirnov values are less than 0.21 except for the Halphen B distribution. The PPCC values are close to 1 and the values of RMSD are less than 1. Moreover, the comparison of the fittings of the four distributions shown in Figure 11, conducts to accept all the distributions for prediction of the probable maximum instantaneous discharges.

Vol. 70 (2024), No. 2
pp. 17–29, ISSN 1802-5420
DOI 10.35180/gse-2024-0109

Table 2. *Goodness-of-fit test of the used distributions*

Test	Gumbel	Gamma	Pearson III	Halphen B
KS	0.06	0.08	0.05	0.68
PPCC	0.99	0.99	0.99	0.99
RMSD	0.40	0.37	0.07	0.18

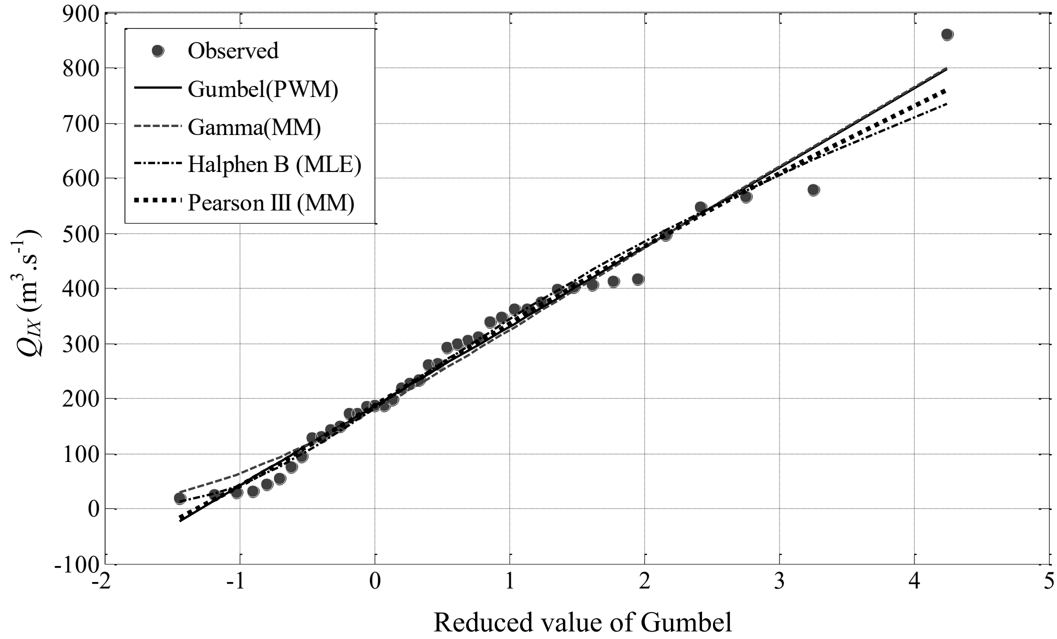

Figure 11. *Comparison of the fittings*

The maximum instantaneous discharges at oued Mazafran using the four distributions are shown in Table 3. In domain of observable frequencies ($T \leq 100$ years) are in the same scale of magnitude, but in rare frequency domain ($T > 100$ years) the discharges by Gumbel and Gamma distributions are higher than the discharges by Pearson Type III and Halphen B distributions.

Table 3. *Maximum instantaneous discharge for different return periods ($m^3.s^{-1}$)*

T (years)	**10**	**20**	**50**	**100**	**500**	**1000**
Gumbel	510	614	748	848	1080	1180
Gamma	510	616	751	850	1074	1168
Pearson Type III	510	604	720	803	987	1063
Halphen B	516	601	700	767	904	957

So, the use Akaike Information Criterion (AIC) [36], makes it possible to select the best distribution. This criterion is expressed by:

$$AIC = n \ln\left(\frac{1}{n-k}\sum_{i=1}^{n}\left(w_i - Q_{IX\,i}\right)^2\right) + 2k \qquad (14)$$

Vol. 70 (2024), No. 2
pp. 17–29, ISSN 1802-5420
DOI 10.35180/gse-2024-0109

Where, w_i is the estimated quantile for the same observed frequency of Q_{IXi} and k is the number of the parameters of the distribution. According to the value of the *AIC* given in Table 4, the lowest value of the *AIC* corresponding to the Gumbel distribution.

Table 4. *Values of AIC*

Type of distribution	*AIC*
Gumbel	273
Gamma	283
Pearson III	298
Halphen B	286

According to the obtaining results, floods can cause an important flooding, for the flood of March 1974 reached to 861 $m^3.s^{-1}$, caused 52 deaths and severely destroyed 4570 houses and 13 bridges [2] close to the 109 years. Hence, the establishment of prevention and protection system against the floods is more necessary.

5 CONCLUSION

Because of the flooding problems related to the extreme floods in the catchment of the oued Mazafran in the North of Algeria, it is very necessary to conduct a study on frequent floods. This study is very important when using geographical information system to locate the areas at risk, also the management of the stream in order to protect the areas against the risk of flooding. To predict the flooding in the catchment, the study focuses on the frequency analysis of maximum instantaneous discharges at Fer a Cheval hydrometric station, well this data must be fitted to a distribution of probability of extremes in hydrology. But, the choice of the distribution to fit the series of maximum instantaneous discharges Q_{IX} is not an easy task because there are quite probability distributions used in the analysis of extreme values. So, the use of the decision support system (DSS) which is based on the tails of extreme event distributions by the use of four graphs conduct finally to the adequate distributions.

The different return-period floods obtained by all the distributions used for the analysis are acceptable and may be considered for simulating water surface profiles and in determining the extent of flooding and management of the catchment of oued Mazafran.

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Vol. 70 (2024), No. 2
pp. 17–29, ISSN 1802-5420
DOI 10.35180/gse-2024-0109