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A STUDY OF THE INFLUENCE OF ECONOMIC FACTORS ON WORLD SILVER PRODUCTION

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ABSTRACT

Silver is a very important raw material used in industry, especially in the electrical and energy industries. With the increase in electromobility, its potential will grow in the future, and any shortage of silver on the world market could be a threat to modern industry. This article deals with the influence of economic factors (price, population, gross domestic product (GDP) and cumulative inflation) on silver production and the creation of appropriate econometric models that best express the relationship between production and economic factors for the period 2000–2020. The influence of economic factors on world silver production is examined using the coefficient of determination and information criteria. The authors use regression analysis in the article, especially these four types: linear, exponential, logarithmic and power. It is evident from the research that the best functional form of regression is exponential according to the coefficient of determination. Based on the investigated economic factors, it has been found that the price is unsuitable for the creation of econometrics; on the other hand, the other factors are eligible.

Keywords: Coefficient of determination; Econometric model; Economic factors; Information criteria; World silver production.

1 INTRODUCTION

Satisfying human needs is inextricably linked to the mineral wealth of our planet. With the same number of people living on earth, the pressure is determined not only by mineral wealth but also by the importance of the many increases. In addition to the rise in the number of inhabitants, their standard of living also increases. The increasing standard of living also means the consumption of goods on a qualitatively and quantitatively different level, which also dynamizes the innovation process. The problem, however, is securing all the necessary resources. Securing mineral resources is a demanding and complicated process requiring a number of decisions, both at the level of the state and at the level of companies operating in the raw materials sector. Decision-making can be characterized as one of the basic managerial activities. Its quality affects the results and efficiency of the entire company. Choosing between several variants can be quite complicated. Therefore, mathematical apparatus (mathematical models, econometric models) and various methods (timely planning, strategic management, price prediction) are used as support tools for managerial decision-making, which help to make the right decisions.

As part of the article, research was conducted on the topic of econometric models in connection with the dependence of economic factors on silver production. The search interval for professional articles was chosen according to the editions for the period 2000–2020. Three stages of research were used for professional publications. The first phase was a preparation in the form of determining suitable keywords (econometric model, economic factors, silver production), then the search itself in the selected Web of Science database (https://www.webofscience.com) and the specification of the search query in the form of appropriately used Boolean AND operators, and the final processing of the relevant results according to the established rules. Only one article was found, titled Forecast Model of Russia's Gross Domestic Product Depending on Financial

Instruments of Trade in Energy and Commodities. The article describes the methodology of forecasting the gross domestic product (GDP) growth for complex socioeconomic systems projected on the economic conditions of the Russian Federation. The most important factors affecting GDP change, the development of a GDP forecast econometric model for the Russian economy and the methodology of the model usage are identified. The model is used as a source of information necessary for analysing the territorial multi-sectoral objects (ATMO) model we developed. ATMO model is the model of planning regional sectoral production systems of independent corporate participants that can predict their behaviour using a game approach. The ATMO model makes it possible to influence the economy to achieve macroeconomic goals and allows for increasing the available volumes of energy resources for export, e.g. for the functioning of the emerging gas hub in the EU. There are no more articles on the topic of econometric models of the dependence of economic factors on production, therefore, there is a gap to create an article. [12–14]

In this, the authors will deal with the study of the effect of economic factors on the production of silver. In the introduction, the basic information about silver and the goals of the article will be described. In the next chapters, the article will deal with materials, methods and results of econometric modelling.

Silver has the chemical marking Ag, a chemical element, a brilliant white metal admired for its decorative beauty and electric conductivity. Silver is located between Group 11 and Period 5 of the Period table between copper (period 4) and gold (period 6). Its physical and chemical properties are intermediates between these two metals.

Silver has the highest thermal conductivity of metals, used to manufacture printed electrical circuits and as a liquid-deposited coating of electronic conductors. It is made of nickel and palladium for electrical contacts. Silver is also used as a catalyst that can transform ethylene into ethylene oxide, the first precursor to many organic compounds. Silver is one of the noblest, i.e., with a lower chemical reaction rate, transition elements.

Silver is widely distributed in nature, but its total quantity is relatively low compared to other metals, accounting for 0.05 parts per million of the Earth’s crust. In fact, all lead, copper, and zinc sulphides contain a small amount of silver. Silver ore may contain amounts of silver ranging from one trace to several thousand troy ounces per ton of avoirdupois, or approximately 10%.

Historically, the main uses of silver were in the form of deposits and coins. However, by the 1960s, demand for industrial silver, particularly in the photographic industry, exceeded world production. In the early 21st century, digital cameras supplanted those that used film, but demand for silver from other sectors—such as for sterling and plated silverware, ornaments, jewellery, coinage, electronic components, and photovoltaic cells—continued to be important.

The article has 3 main goals:

1. To what extent do econometric models of the dependence of world silver production on selected economic factors meet the prerequisites for their proper functioning?
2. Which type of regression best describes the dependence of silver production on the selected economic factors?
3. Which economic factor most affects silver production? [3]

The article also formulates 2 hypotheses.

Hypothesis H1: The least important determinant of world silver production is its price.
Hypothesis H2: The fundamental determinant of world silver production is the world’s population.

2 MATERIALS A METHOD

To fulfil the goals of the article, an input data matrix was created (Table 1). Econometric models of the dependence of world silver production on selected economic factors were obtained from data for the period 2000 to 2020. World silver production is given in metric tons and represents an explained (dependent) variable from the point of view of econometric modelling. It is a variable with an increasing trend throughout the examined period and slight decreases in 2006, 2015, 2017, 2018, 2019 and 2020. All other variables representing selected economic factors are so-called explanatory (independent) variables that influence the explained variable.

The macroeconomic findings show that demand (or world production) for any raw material depends not only on its price but also on the population, the GDP, the price of other products and services and other economic factors [1].

The first economic factor on which the article will focus our attention is the price of the raw material. It is generally known the demand, and therefore the supply, of the raw material on the world market is influenced by its price [7]. With increasing demand, the price of raw materials rises, and with decreasing demand, the opposite is true. However, other factors are now increasingly influencing the price. Many speculators demand more raw materials than would correspond to the estimated real demand for this raw material. There are also purely political motives for purchasing raw materials – greater diversification of sources, particularly in the case of imports of raw materials from conflict or potentially conflict areas. For the above-mentioned reasons, the H1 hypothesis, i.e. that the price of silver has the least influence on world silver production, has been formulated.

Another economic factor is the growth of the world's population, which certainly has a very positive effect on the growth of supply and demand for raw materials. The same is true for world wealth growth, as measured by GDP per capita [7]. Both these factors certainly have a positive and long-term impact on world production and demand for raw materials. However, in connection with the above factors, it is necessary to mention the impact of technological progress that aims to permanently reduce the share of raw materials in tangible products [4][6]. Also, the increased use of recycling raw materials from end-of-life products has a positive effect on reducing the production of raw materials. It follows that at a certain level of marginal growth of the world's population and the wealth of the population, the world production of raw materials may no longer immediately increase.

The fourth economic factor, also closely related to the demand or supply of raw materials, is inflation. The sharp growth in demand for raw materials has a positive effect on annual inflation and, thus, on cumulative inflation [7]. However, unlike the previous factors, this is an indirect factor caused by the interaction of demand and the supply of raw materials in the world. For the above-mentioned reasons, the H2 hypothesis, i.e. that the world's population has the greatest influence on world silver production, was formulated.

In the case of the price of silver, stated in US$/t, it is an economic factor with an increasing trend as well, but the development of this variable is very volatile. Periods of price growth and price decline alternate. The highest price increase was recorded in 2011 (by 484,190 US$/t), and the lowest decrease (by 235,344 US$/t) in 2013. [3]

Another economic factor (explanatory variable) affecting world silver production is undoubtedly world population. In the entire investigated period, this is a purely increasing monotonic function. [3]

The third economic factor is world GDP per capita in US$. The time trend of this economic factor is increasing – from 2000 to 2020, the value of this factor has almost doubled. However, in 2001, 2009, 2015 and 2020, there was a decrease in this factor. [3]

The fourth economic factor is inflation, monitored year-on-year and cumulatively. The value of world cumulative inflation is derived from year-on-year world inflation so that at the beginning of the examined period in 2000, it has a value of 100, which is gradually adjusted annually by year-on-year inflation. Year-on-year world inflation expressed in % is very volatile throughout the monitored period, but there was no deflation, so all values are positive. The highest world year-on-year inflation was recorded in 2008 at around 8.95% and the lowest in 2015 at around 1.43%. World cumulative inflation, as follows from the above, represents a purely increasing monotonic function. [3]

Table 1. *Input data matrix for the implementation of econometric models of the dependence of world silver production on selected economic factors*

Year	World production (mt)	Economic factors				
		Price (US$/t)	World population (people)	GDP per capita (PPP)(US$)	World inflation (year-on-year in %)	Cumulative inflation (%)
2000	18 100	160 754	6143493823	5512.217025	3.494	100
2001	18 700	141 142	6222626606	5407.11511	3.838	103.8382846
2002	18 800	148 536	6301773188	5541.909015	2.982	106.9343005
2003	18 800	157 860	6381185114	6138.104723	3.032	110.1766976
2004	20 000	215 088	6461159389	6829.414125	3.383	113.9035862
2005	20 800	235 986	6541907027	7305.393939	4.112	118.5875111
2006	20 100	373 270	6623517833	7818.995169	4.282	123.665898
2007	20 800	431 785	6705946610	8701.177804	4.817	129.622597
2008	21 300	482 904	6789088686	9430.586423	8.953	141.2276695
2009	22 300	472 294	6872767093	8837.219447	2.936	145,3744409
2010	23 300	649 445	6956823603	9558.77174	3.347	150.2401234
2011	23 300	1 133 635	7041194301	10494.83677	4.805	157.4591613
2012	24 300	1 003 425	7125828059	10609.77086	3.685	163.2615314
2013	26 700	768 081	7210581976	10784.66484	2.594	167.4965356
2014	27 800	613 758	7295290765	10951.8086	2.35	171.4327042
2015	27 500	505 410	7379797139	10223.00396	1.432	173.8876205
2016	27 900	552 993	7464022049	10267.63884	1.503	176.5011514
2017	27 800	549 135	7547858925	10801.43975	2.177	180.3435815
2018	27 000	505 731	7631091040	11345.34998	2.439	184.7421614
2019	26 500	661 662	7713468100	11394.86113	2.137	188.6901014
2020	23 500	803 769	7794798739	10909.28705	1.94	192.3506894

Sources: [5][9][10][11]

An econometric model transforms an economic model into a form that can be analysed with the help of econometric tools and techniques. Economic relationships are not usually exact. The economic theory to systematically predict the behaviour of individuals or companies rather describes average or large numbers of companies or people. A simple example of an econometric model assumes that monthly consumer spending is linearly dependent on consumer income in the previous month. Different types of regression analysis are used to determine regression coefficients. It depends on the relationship between the explained and the explanatory variable. Therefore, we distinguish between linear and non-linear types of regression. The article will use a regression model with several types – linear, exponential, logarithmic, and power. The results of the article will interpret the appropriate regression type. [2]

1. The linear regression $y = b_0 + b_1 . x$ ***(1)***
2. The exponential regression $\ln(y) = \ln(b_0) + b_1 . x$ ***(2)***
3. The logarithmic regression $y = b_0 + b_1 . \ln(x)$ ***(3)***
4. The power regression $\ln(y) = \ln(b_0) + b_1 . \ln(x)$ ***(4)***

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2.1 A simple linear regression model

$$Y_t = \beta_1 + \beta_2 . X_t + u_t , t = 1, 2, ..., n \quad \textbf{(5)}$$

where β_1 is the regression parameter of the level constant; β_2 is the regression slope parameter; Y_t represents the explained (dependent) variable; X_t is the explanatory (independent) variable; t = 1, 2, …, n represents the investigated period. In this article, Y_t represents world silver production, and X_t one of the following economic factors examined: average annual silver price, world population, world GDP/capita, and year-on-year or cumulative inflation; u_t is a random component.

The functional form of regression models is important for the correct expression of the dependence of world silver production on selected economic factors. Regression models can be defined according to linear regression, as mentioned above, but also in the form of non-linear regression. Non-linear regressions are also used in our article to fulfil the second goal of the article. The investigated non-linear regression models include logarithmic, exponential, and power regression models. [8]

2.2 A simple logarithmic regression model

This model is defined as follows:

$$Y_t = \beta_1 + \beta_2 . \ln X_t + u_t, t = 1, 2, ..., n \quad \textbf{(6)}$$

where the meaning of the variables and regression parameters is the same as in the previous models. [8]

2.3 A simple exponential regression model

The model is established as follows:

$$\ln Y_t = \ln\beta_1 + \beta_2 . X_t + u_t, t = 1, 2, ..., n \quad \textbf{(7)}$$

where the meaning of the variables and regression parameters is the same as in the previous models. [8]

2.4 A simple power regression model

The exponential simple regression model is expressed as follows:

$$\ln Y_t = \ln \beta_1 + \beta_2 . \ln X_t + u_t, t = 1, 2, ..., n \quad \textbf{(8)}$$

where the meaning of the variables and regression parameters is the same as in the previous models.

All the simple econometric models examined in the article using various functional forms created according to the regressions mentioned above are listed in the following table, including designations and definitions. [8]

Table 2. *Investigated simple econometric models of dependence of world silver production on selected economic factors*

Econometric model	Formulation of a simple econometric model of world silver production
The linear regression models	
EMLIN1	$Production_t = \beta_1 + \beta_2 . Price_t + u_t$
EMLIN2	$Production_t = \beta_1 + \beta_2 . Population_t + u_t$
EMLIN3	$Production_t = \beta_1 + \beta_2 . GDP_t + u_t$
EMLIN4	$Production_t = \beta_1 + \beta_2 . Cumulative\_inflation_t + u_t$
The logarithmic regression models	
EMLOG1	$Production_t = \beta_1 + \beta_2 . \ln Price_t + u_t$
EMLOG2	$Production_t = \beta_1 + \beta_2 . \ln Population_t + u_t$
EMLOG3	$Production_t = \beta_1 + \beta_2 . \ln GDP_t + u_t$
EMLOG4	$Production_t = \beta_1 + \beta_2 . \ln Cumulative\_inflation_t + u_t$
The exponential regression models	
EMEXP1	$\ln Production_t = \ln \beta_1 + \beta_2 . Price_t + u_t$
EMEXP2	$\ln Production_t = \ln \beta_1 + \beta_2 . Population_t + u_t$
EMEXP3	$\ln Production_t = \ln \beta_1 + \beta_2 . GDP_t + u_t$
EMEXP4	$\ln Production_t = \ln \beta_1 + \beta_2 . Cumulative\_inflation_t + u_t$
The power regression models	
EMPOW1	$\ln Production_t = \ln \beta_1 + \beta_2 . \ln Price_t + u_t$
EMPOW2	$\ln Production_t = \ln \beta_1 + \beta_2 . \ln Population_t + u_t$
EMPOW3	$\ln Production_t = \ln \beta_1 + \beta_2 . \ln GDP_t + u_t$
EMPOW4	$\ln Production_t = \ln \beta_1 + \beta_2 . \ln Cumulative\_inflation_t + u_t$

Source: own research

2.5 Tests of the assumptions of the correct functioning of econometric models

To fulfil the first goal of the work, it is necessary to perform a whole series of tests to test the assumptions for the correct functioning of econometric models. The tests are the following: a test of normality of the residual component, a test of the mean value of the residual component, a test of significance of the econometric model, a test of significance of all regressors of the econometric model, a test of correct specification (RESET) of the econometric model, and a test of homoscedasticity and autocorrelation test. [8]

2.5.1 Test of normality of the residual component

Limit basic assumptions of the classical linear regression model is a normal distribution of the random component for all t, which can be expressed as:

$$u \approx N(0, \sigma^2 * I_n) \qquad \textbf{\textit{(9)}}$$

The assumption of normality of the random components is used especially in the specification of the probability distribution of the residual component and the subsequent testing of hypotheses in the model as well as the construction of confidence intervals. In the regression model, the asymptotic results are identical to the results derived so far, for example, the estimation of the regression coefficients by the method of least squares, which behaves according to the normal distribution:

$$\hat{\beta} \sim N(\beta, \sigma^2 (X_1^{'} X_1)^{-1} \qquad \textbf{\textit{(10)}}$$

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When testing the normality of the residual component, we use graphic tools in the form of a histogram of the frequency distribution of the residuals, which we compare with the Gaussian theoretical curve and probabilistic P-P and Q-Q graphs.

The most used non-parametric tests for the normality of residuals include the goodness-of-fit test, the Jarque-Bera test (JB-test) and the Kolmogorov-Smirnov test (KS-test). [8]

2.5.2 Mean residual component test

The test can be used if the population follows a normal distribution (so normality is satisfied), so it is a parametric test. The test is used to verify the hypothesis that the mean value of the population is equal to a specific value, i.e.:

$$\mu = \mu_0 \quad \textbf{\textit{(11)}}$$

2.5.3 Significance test of the econometric model

We will deal with testing the statistical significance of the model, especially the coefficient of determination. When testing the above hypotheses, the Fisher-Snedecor F-distribution is used, and therefore we call this testing the F-test. We obtain the test statistic based on the equation:

$$F_{vyp} = \frac{ESS/df_1}{RSS/df_2} = \frac{ESS/(k-1}{RSS/(n-k)} \sim F(df_1, df_2) \quad \textbf{\textit{(12)}}$$

2.5.4 Significance tests of all regressors of the econometric model

To verify the significance of the parameters, the t-test is used, with the help of which we can test the null hypothesis of individual parameters $H_0: b_j = 0$, where the null hypothesis says that the tested parameter is insignificant. [8]

The test statistic is of the form:

$$t = \frac{b_j - \beta_j}{\sigma_j} \overset{H_0}{\sim} t_{T-1} \quad \textbf{\textit{(13)}}$$

where $b_j$is an estimate of the parameter value, β_j the hypothetical value is equal to zero, σ_j the standard deviation of the estimate, and T the number of observations.

2.5.5 Test of the correct specification (RESET) of the econometric model

RESET test is used to diagnose specification errors that may have arisen due to incorrect functional form specification or by omitting relevant explanatory variables. [8]

2.5.6 Test of homoscedasticity

The opposite of homoscedasticity is the changing variance of the random component, so-called heteroscedasticity. The variance dependence of the random component can generally take a different functional form depending on the explanatory variable. Heteroscedasticity is a case of violation of the assumption of homoscedasticity. In other words, if the residual components do not have constant variance, then they are referred to as heteroskedastic. [2]

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2.5.7 Autocorrelation test

Autocorrelation of residuals violates the classical assumption of a linear regression model. The residual component ε is correlated with its lagged and future values. Autocorrelation occurs primarily in time series.

2.6 Evaluating econometric models of the dependence of world silver production on economic factors

To fulfil the second and third objectives, the authors will use the coefficient of determination R^2 and information criteria.

2.6.1 Coefficient of determination R^2

The coefficient of determination expresses the degree of explanation of the overall change of the explained variable Y by regression, i.e. by the action of the linear relationship of the explanatory variable. It is, therefore, a criterion of agreement between observed data and estimates by means of a regression line. It ranges from zero to one. If its value equals 1, all sample observations lie directly on the smoothed regression line, i.e. the best degree of smoothing. If equal to 0, it means that no observation lies on the estimated sample regression line, and all information remains unexplained in the residual. In this case, the estimated regression model is meaningless. In practice, the coefficient of determination is interpreted in such a way that its value expresses the degree of variability of the explained variable Y around its average value by the regression model.

The coefficient also has its shortcomings that consist in the fact that it does not adequately respond to changes in the number of observations in the regression model and does not consider the expansion of the number of explanatory variables in the regression model. The coefficient of determination is available in SPSS. [13, 15]

2.6.2 Information criteria

As more variables are added to the regression model, the accuracy of the estimate may increase, but the risk of overestimating the model also increases. These criteria solve the problem that the corrected coefficient of determination does not sufficiently penalize the increasing number of explanatory variables when assessing the choice of explanatory variables.

These criteria are derived from information theory, and the values of these criteria usually depend on the number of regressors $\boldsymbol{k}$ and the number of observations $\boldsymbol{n}$, and the maximum likelihood estimate of the variance of the residual component $\boldsymbol{s_u^2(k)}$ depending on **k**. The most important information criteria include *Akaike Information Criterion (*AIC), *Bayes Information Criterion* (BIC), and *Hannan and Quinn Criterion* (HQ). When choosing, we look for a minimum information criterion. In this article, the minimum sum of the values of all three criteria was used for the evaluation of econometric models.

The **AIC** criterion is given by the relation:

$$AIC(k) = ln\left(s_u^2(k)\right) + \frac{2k}{n}. \qquad \textbf{\textit{(14)}}$$

Where ask increases, the first term decreases and the second term penalizes a larger number of explanatory variables.

The **BIC** criterion is given by:

$$BIC(k) = ln\left(s_u^2(k)\right) + \frac{kln(n)}{n}. \qquad \textbf{\textit{(15)}}$$

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For values n≥10, it is penalizing, i.e. the second term in the equation is higher than that of AIC, and therefore this criterion selects smaller (simpler) models than AIC.

The **HQ** criterion is given by:

$$BIC(k) = \ln\left(s_u^2(k)\right) + \frac{2kln(\log(n))}{n} \qquad \textbf{\textit{(16)}}$$

This criterion is in the intensity of the growth penalty ***k*** between AIC and BIC. All information criteria are part of econometric software products. [8]

3 RESULTS

The results of the implementation of the first goal of the article (Table 3) clearly show that none of the 16 examined econometric models created using linear, logarithmic, exponential, and power regression passed the 7 tests for meeting the prerequisites for the correct functioning of these models by the autocorrelation test. The main cause of autocorrelation can be inertia in the development of economic variables. Most macroeconomic time series show inertia in their long-term development, and values over time are strongly dependent on previous values.

Another important test for the correct functioning of econometric models is the homoscedasticity test. The test for the existence of homoscedasticity (constant variance) of the residual component was not confirmed for 8 econometric models. Two of them were created using linear regression (EMLIN2, EMLIN4), which includes the world population and cumulative inflation as explanatory variables, one model built by logarithmic regression (EMLOG1) containing the explanatory variable silver price, two models created by exponential regression (EMEXP2, EMEXP4), in which world population and cumulative inflation appear as explanatory variables, and finally, three models built according to power regression (EMPOW1, EMPOW2, EMPOW4), which include silver price, world population and cumulative inflation as explanatory variables. It follows from the above that all econometric models that include GDP as an explanatory variable do not show problems with heteroscedasticity. The cause of heteroscedasticity can be, e.g., outlier observations, the incorrect regression model, etc.

The third test that some econometric models fail to pass is the normality test of the residual component. It was the case for three models, two of which were created by linear regression (EMLIN1, EMLIN2) and include silver price and world population as explanatory variables, and one model built by exponential regression (EMEXP2) which includes world population as an explanatory variable.

The last fourth test, which was unsuccessful for some econometric models, is the test of correct specification of the model (RESET). In the case of two models, one built by linear regression (EMLIN1) and the other one using power regression (EMPOW1), both of which work as an explanatory variable with the price of silver, misspecification was detected. A bad model specification is due to a bad form or a missing essential explanatory variable. [8]

All other tests, such as the test of the mean value of the residual component, the test of significance of the model, and the test of significance of all regressors, were fulfilled for all 16 econometric models.

The best econometric models from the point of view of fulfilling the assumptions for their proper functioning include a total of seven models, one of which consists of linear regression (EMLIN3), three models using logarithmic regression (EMLOG2, EMLOG3, EMLOG4), two models using exponential regression (EMEXP1, EMEXP3) and one model via power regression (EMPOW3). All of the above models meet all the assumptions for proper functioning except for autocorrelation. The results also clearly show that all econometric models containing the explanatory variable GDP belong to this group. On the contrary, the worst models, from the point of view of fulfilling the prerequisites for their proper functioning, belong to those models that meet only four prerequisites. There is a total of four econometric models, two of which are created by linear regression (EMLIN1, EMLIN2), one by exponential regression (EMEXP2) and one by power regression (EMPOW1).

Table 3. *Testing simple econometric models of dependence of world silver production on selected economic factors*

Tests	Simple regression models			
The linear regression models				
	EMLIN1	EMLIN2	EMLIN3	EMLIN4
normality of residuals	it is rejected	it is rejected	it is not rejected	it is not rejected
mean value of residuals (zero)	it is not rejected	it is not rejected	it is not rejected	it is not rejected
significance of the model	it is not rejected	it is not rejected	it is not rejected	it is not rejected
significance of all regressors	it is not rejected	it is not rejected	it is not rejected	it is not rejected
correct model specification (RESET)	it is rejected	it is not rejected	it is not rejected	it is not rejected
homoscedasticity (exists)	it is not rejected	it is rejected	it is not rejected	it is rejected
autocorrelation (not significant)	it is rejected	it is rejected	it is rejected	it is rejected
The logarithmic regression models				
	EMLOG1	EMLOG2	EMLOG3	EMLOG4
normality of residuals	it is not rejected	it is not rejected	it is not rejected	it is not rejected
mean value of residuals (zero)	it is not rejected	it is not rejected	it is not rejected	it is not rejected
Significance of the model	it is not rejected	it is not rejected	it is not rejected	it is not rejected
significance of all regressors	it is not rejected	it is not rejected	it is not rejected	it is not rejected
correct model specification (RESET)	it is not rejected	it is not rejected	it is not rejected	it is not rejected
homoscedasticity (exists)	it is rejected	it is not rejected	it is not rejected	it is not rejected
autocorrelation (not significant)	it is rejected	it is rejected	it is rejected	it is rejected
The exponential regression models				
	EMEXP1	EMEXP2	EMEXP3	EMEXP4
normality of residuals	it is not rejected	it is rejected	it is not rejected	it is not rejected
mean value of residuals (zero)	it is not rejected	it is not rejected	it is not rejected	it is not rejected
significance of the model	it is not rejected	it is not rejected	it is not rejected	it is not rejected
significance of all regressors	it is not rejected	it is not rejected	it is not rejected	it is not rejected
correct model specification (RESET)	it is not rejected	it is not rejected	it is not rejected	it is not rejected
homoscedasticity (exists)	it is not rejected	it is rejected	it is not rejected	it is rejected
autocorrelation (not significant)	it is rejected	it is rejected	it is rejected	it is rejected
The power regression models				
	EMPOW1	EMPOW2	EMPOW3	EMPOW4
normality of residuals	it is not rejected	it is not rejected	it is not rejected	it is not rejected
mean value of residuals (zero)	it is not rejected	it is not rejected	it is not rejected	it is not rejected
significance of the model	it is not rejected	it is not rejected	it is not rejected	it is not rejected
significance of all regressors	it is not rejected	it is not rejected	it is not rejected	it is not rejected

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correct model specification (RESET)	it is rejected	it is not rejected	it is not rejected	it is not rejected
homoscedasticity (exists)	it is rejected	it is rejected	it is not rejected	it is rejected
autocorrelation (not significant)	it is rejected	it is rejected	it is rejected	it is rejected

Source: own research

The results of the implementation of the second goal of the article (Table 4), both according to the coefficient of determination and according to the minimization of the sum of information criteria, show that the best values are recorded for econometric models created with the help of exponential regression (EMEXP1, EMEXP2, EMEXP4) only in the case of the EMEXP3 model, where the explanatory variable is GDP, this regression form ranked second behind the EMPOW3 model, which is built by power regressions.

Conversely, the worst results were identified for linear regression (EMLIN1, EMLIN2, EMLIN4), except for the EMLIN3 model, which includes GDP as an explanatory variable, which ranked third, and the worst was the EMLOG3 model created using logarithmic regression.

For the remaining forms of regression, such as logarithmic regression and power regression, the results are diverse. In the case of econometric models with world population and cumulative inflation, the logarithmic regression ranked third, and the power regression ranked second for the econometric model with the price of silver, which was exactly the opposite.

The results of the implementation of the third objective (Table 4) clearly show that the greatest influence on world silver production according to all regression forms is cumulative inflation and, conversely, the lowest silver price. According to linear, logarithmic, and exponential regression forms, the second largest influence on world silver production is world population, and the third one is GDP/capita. In the case of the power regression form, the order of second and third place is reversed. The results of the implementation of the third objective of the article clearly show that the H1 hypothesis about the smallest influence of the silver price on world silver production has been confirmed. Hypothesis H2 about the largest influence of the world population on world silver production has not been confirmed. However, it should be noted that cumulative inflation is an indirect factor that accompanies a change in the demand or supply of raw materials. It follows that the world population of direct factors most affects world silver production based on all regression forms performed except the power regression form.

Factors such as world population, GDP per capita and cumulative inflation are shown to have a significant impact on world silver production. Based on econometrics, the modelling showed that price as an economic factor, according to the coefficient of determination, insufficiently explains the impact on the total world silver production between 2000 and 2020. The authors believe that given the stable growth of silver production in the world, the growth of economic factors such as world population, GDP per capita and cumulative inflation differences will be minimal. According to the research results, the authors concluded that the average annual world price of silver, given its volatility, does not adequately explain its impact on world silver production. Therefore, we do not recommend its use in one-dimensional econometric models for the development prediction of world silver production. On the other hand, all econometric models explaining the impact of other factors studied on world silver production can be used practically to predict it.

Table 4. *Evaluation of simple econometric models of dependence of world silver production on selected economic factors according to information criteria and coefficient of determination*

Econometric model	Criterion AIC(k)	Criterion BIC(k)	Criterion HQ(k)	Total information criteria	Coefficient of determination (R^2)
The linear regression models					
EMLIN1	16,05331198	16,15272661	15,8945511	48,10058969	0,371708211
EMLIN2	14,84430898	14,94372361	14,68554811	44,4735807	0,812458212
EMLIN3	14,90931996	15,00873459	14,75055909	44,66861364	0,79986089
EMLIN4	14,65589945	14,75531408	14,49713857	43,90835209	0,844663744
The logarithmic regression models					
EMLOG1	15,74778531	15,84719994	15,58902443	47,18400967	0,537115291
EMLOG2	14,79504558	14,89446021	14,6362847	44,32579049	0,821473278
EMLOG3	15,0488446	15,14825923	14,89008373	45,08718757	0,769894635
EMLOG4	14,62837441	14,72778904	14,46961353	43,82577698	0,848881073
The exponential regression models					
EMEXP1	-4,442000125	-4,342585496	-4,600760998	-13,38534662	0,583964337
EMEXP2	-5,411633887	-5,312219258	-5,57039476	-16,2942479	0,842
EMEXP3	-5,22416018	-5,124745551	-5,382921053	-15,73182678	0,809680721
EMEXP4	-5,645994801	-5,546580172	-5,804755674	-16,99733065	0,875
The power regression models					
EMPOW1	-4,0928101	-3,99339547	-4,251570972	-12,33777654	0,41
EMPOW2	-5,347170229	-5,2477556	-5,505931102	-16,10085693	0,831
EMPOW3	-5,36788574	-5,268471111	-5,526646613	-16,16300346	0,835
EMPOW4	-5,574020815	-5,474606185	-5,732781687	-16,78140869	0,866

Source: own research

4 CONCLUSION

Research results demonstrably show that in the period from 2000 to 2020, there is a very strong relationship between world silver production and economic factors such as population, world GDP per capita in purchasing power parity and cumulative inflation. Econometric models that express this relationship, regardless of the type of regression we used, reach a very high value of the coefficient of determination. Although for the next period, the econometric models would have slightly different regression coefficients given the stability of these economic factors and world silver production, these factors explain this production quite well. Simple econometric models are recommended for short- and medium-term predictions and their updating regularly after this time. The research also clearly showed that the average annual world silver price does not sufficiently explain the impact on world silver production since econometric models expressing the effect of the world silver price on silver production show a very low coefficient of determination. Therefore, they are not suitable for prediction. The authors believe this is mainly due to price fluctuations over time caused by non-economic factors such as political interests, lobbying interests and other factors. The article will have several benefits. The first is the contribution to science when econometric models of selected mineral resources that have not yet been developed will be created and will be a model for processing other mineral resources. Another is the practical benefit when the developed econometric models will be possible for company managers to use in strategic management. The work results can be used for creating industrial policy proposals and monitoring the state and performance of individual mineral resources. Research can continue with new data.

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