COMPARISON OF A MULTIPLE REGRESSION MODEL AND A TWO-REGIME MODEL OF A VERTICAL REFRACTION

POROVNANIE VIACNÁSOBNÉHO REGRESNÉHO MODELU A DVOJREŽIMOVÉHO MODELU VERTIKÁLNEJ REFRAKCIE

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Abstract

Accuracy of the trigonometric measurement of elevations is affected by the systematic influence of a vertical refraction, which is caused by changes of meteorological parameters. Submitted paper deals with a modelling of the impact of the vertical refraction using selected meteorological parameters. At first, a concise derivation of a physical principle of the vertical refraction is given. Then, a multiple regression model and its extension into a form of two-regime model are given. Division into two regimes provides a threshold function, which expresses the dependence of the original explanatory variables. Different types of the threshold function are considered and finally a comparison of the quality of the proposed models and application of a chosen model on the results of repeated trigonometric measurements is given.

Abstrakt

Presnosť trigonometrického merania prevýšení ovplyvňuje vplyv vertikálnej refrakcie, ktorá je spôsobená zmenami meteorologických prvkov. Príspevok sa zaoberá modelovaním vplyvu vertikálnej refrakcie pomocou vybraných meteorologických prvkov. Úvodná časť sa venuje stručnému odvodeniu fyzikálneho princípu vertikálnej refrakcie. Následne je definovaný viacnásobný regresný model vertikálnej refrakcie, ktorý je ďalej rozvinutý do dvojrežimového modelu. Rozdelenie do dvoch režimov zabezpečuje prahová funkcia, ktorá vyjadruje závislosť pôvodných vysvetľujúcich premenných. Rôzne typy prahovej funkcie sú uvážené a následne porovnanie navrhnutých modelov ako aj použitie vybraného modelu je uvedené.

Keywords: trigonometric measurement of elevation, vertical refraction, refractive index, meteorological parameters, regression model, two-regime model, threshold function

1 INTRODUCTION

The results of surveying measurements are affected by the measurement errors that arise due to imperfections in instruments, human senses and due to influence of an environment. Because the rapid progress in various sectors (sensing devices and sensors, computing technology, transmission and processing of data) for the past two decades has positively reflected into the field of instrumental and personal errors, a topic of the environment influence still represents open and creative space.

The intensity and trajectory of the propagating light rays through the atmosphere is affected by absorption, diffusion, diffraction, reflection and refraction. The trigonometric measurement of elevations is mainly affected by a vertical part of the refraction, i. e. vertical refraction, which irrespective of the current modern geodetic instruments remains a limiting factor of using this method [4], [8], [16].

2 PHYSICAL MODEL OF THE VERTICAL REFRACTION

According to [17], vertical refraction is defined as a curvature of the light rays transmitting from a source to a receiver caused by an unstable density of the air layers. Its consequence, an observer sights a target in the direction of a tangent to a spatially curved path from the point of observation (fig. 1).

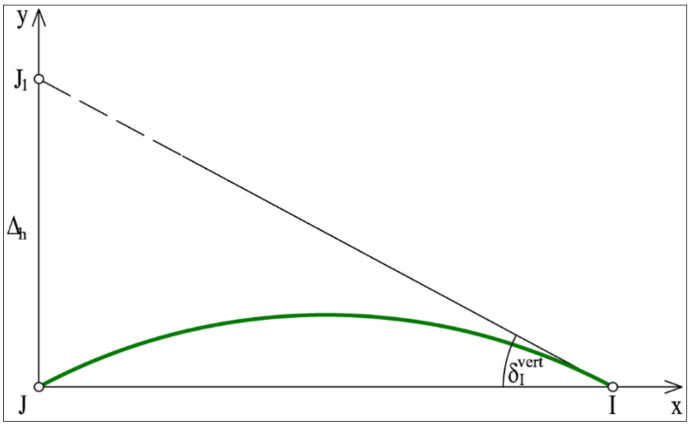

Fig. 1 Influence of the vertical refraction

A deviation in the measured elevation, caused by the influence of the vertical refraction, can be determined by the relationship [10]:

$$\Delta_h = -\frac{1}{n_{gr}^{J}} \cdot \int_0^{x_I} x \cdot grad_h n_{gr} \cdot \sin\left(z_{IJ}^{m}\right) \; dx\,, \tag{1}$$

where n_{gr} is the group refractive index of air, $grad_h n_{gr}$ – the vertical gradient of n_{gr} and z_{IJ}^{m} – the measured zenith angle. The group refractive index of air depends on a wavelength of the electromagnetic waves, physical conditions of an environment and its chemical composition. Changes of the refractive index are mainly related to the changes of basic meteorological parameters – the air temperature, humidity and pressure. In the field of visible light describes this dependence empirical Barrel – Sears relationship [15]:

$$(n_{gr}-1)\cdot 10^6 = N_{gr} = \left(\frac{T_0}{p_0}\cdot N_{gr}^{0}\cdot\frac{p}{T}\right) - 11{,}27\cdot\frac{e}{T}\,, \tag{2}$$

where N_{gr} is the group refractivity, N_{gr}^{0} – the group refractivity of a standard atmosphere, T_0, p_0 – the temperature in [K] and the pressure in [hPa] of a standard atmosphere, T, p, e – the temperature, the pressure and the water vapor pressure in [hPa] of an ambient atmosphere. Considering that the meteorological parameters are a function of height, the vertical gradient of the refractive index can be expressed as follows:

$$\frac{\partial n_{gr}}{\partial h} = \frac{\partial n_{gr}}{\partial T}\cdot\frac{\partial T}{\partial h} + \frac{\partial n_{gr}}{\partial p}\cdot\frac{\partial p}{\partial h} + \frac{\partial n_{gr}}{\partial e}\cdot\frac{\partial e}{\partial h}\,, \tag{3}$$

where

$$\frac{\partial n_{gr}}{\partial T} = \left(-82{,}23\cdot\frac{p}{T^2} + 11{,}27\cdot\frac{e}{T^2}\right)\cdot 10^{-6}\,,\quad \frac{\partial n_{gr}}{\partial p} = \frac{82{,}23}{T}\cdot 10^{-6}\,,\quad \frac{\partial n_{gr}}{\partial e} = -\frac{11{,}27}{T}\cdot 10^{-6}\,. \tag{4}$$

Ignoring the impact of the changes of the water vapour pressure and omitting the contribution of the second member of the refractive index gradient with respect to the temperature, equation (1) can be written as follows [14]:

$$\Delta_h = \frac{1}{n_{sk}^{J}} \cdot \int_0^{x_I} x \cdot \left\{ \left(82{,}23\cdot\frac{p}{T^2}\cdot\left(\frac{\partial T}{\partial h} - \frac{T}{p}\cdot\frac{\partial p}{\partial h}\right)\right)\cdot 10^{-6}\cdot\sin\left(z_{IJ}^{m}\right)\right\} \; dx\,. \tag{5}$$

The above mentioned equation can be further simplified by expressing the dependency of the atmospheric pressure on a height. On the elementary volume of air acts downward the gravity dG and the buoyancy force dF in the opposite direction (fig. 2).

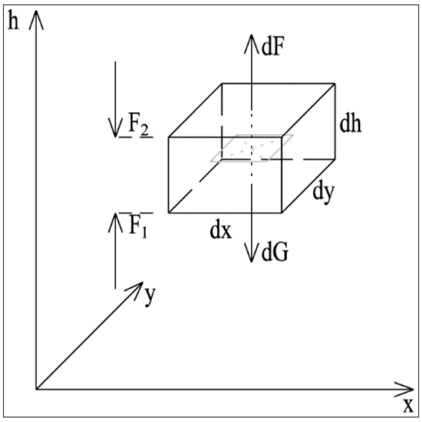

Fig. 2 The gravity and buoyancy force

The buoyancy force is equal to the difference of two pressure forces F_1 and F_2 acting in the vertical plane. Let p is the pressure acting on the lower base, then force F_1 equals to:

$$F_1 = p \cdot dx \cdot dy\,, \tag{6}$$

while force F_2, acting on the upper base, equals to:

$$F_2 = \left(p + \frac{\partial p}{\partial h} \cdot dh \right) \cdot dx \cdot dy\,. \tag{7}$$

Then, for the buoyancy force can be written [2]:

$$dF = F_1 - F_2 = p \cdot dx \cdot dy - p \cdot dx \cdot dy - \frac{\partial p}{\partial h} \cdot dx \cdot dy \cdot dh = -\frac{\partial p}{\partial h} \cdot dV\,. \tag{8}$$

The gravity equals to:

$$dG = dm \cdot g = \rho \cdot dV \cdot g \tag{9}$$

and subsequently, in pursuance of the condition of the balance between forces $dF = dG$, can be for the vertical pressure gradient written:

$$\frac{\partial p}{\partial h} = -g \cdot \rho\,, \tag{10}$$

where g is the gravitational acceleration ($g = 9{,}81\ \mathrm{m \cdot s^{-2}}$) and ρ is the density of air. The density of air depends on the air temperature and pressure and can be determined from the ideal gas law [18]:

$$p \cdot V = n \cdot R \cdot T\,, \tag{11}$$

where V is the volume of a gas, n – the amount of a substance of a gas, which is:

$$n = \frac{m}{M}\,, \tag{12}$$

m – the mass of a gas, which is associated with the density of a gas according to the following:

$$m = \rho \cdot V\,, \tag{13}$$

M – the molar mass of a gas, i.e. the mass of one mole of a substance,

R – the molar gas constant ($R = 8{,}314\ \mathrm{J \cdot K^{-1} \cdot mol^{-1}}$).

Based on the equations (11), (12) and (13) can be for the density of a gas written:

$$\rho = \frac{p \cdot M}{R \cdot T} \tag{14}$$

and analogically, for the density of a gas ρ_0 under the conditions T_0 and p_0:

$$\rho_0 = \frac{p_0 \cdot M}{R \cdot T_0}\,. \tag{15}$$

Volume LXI (2015), No.1
p. 32-41, ISSN 1802-5420

If $T_0 = 273{,}15\,\mathrm{K}$ and $p_0 = 1013{,}25\,\mathrm{hPa}$, the density of air equals to $\rho_0 = 1{,}29\ \mathrm{kg \cdot m^{-3}}$. Dividing equation (14) by equation (15), for the density of air is obtained:

$$\rho = \rho_0 \cdot \frac{T_0}{p_0} \cdot \frac{p}{T}. \tag{16}$$

Substituting equation (16) into equation (10), the vertical pressure gradient equals to:

$$\frac{\partial p}{\partial h} = -g \cdot \rho_0 \cdot \frac{T_0}{p_0} \cdot \frac{p}{T} = -0{,}034 \cdot \frac{p}{T}. \tag{17}$$

Finally, combining equations (5) and (17), an equation representing the physical model of the vertical refraction impact can be written in the following form [14]:

$$\Delta_h = \frac{1}{n_{sk}^{\mathrm{J}}} \cdot \int_0^{x_1} x \cdot \left\{ \left(82{,}23 \cdot \frac{p}{T^2} \cdot \left(\frac{\partial T}{\partial h} + 0{,}034 \right) \right) \cdot 10^{-6} \cdot \sin\left(z_{\mathrm{IJ}}^m\right) \right\} \, dx. \tag{18}$$

The above mentioned derivation shows, that the physical principle of the vertical refraction depends on the knowledge of the light wavelength, on the meteorological parameters of the atmosphere ground layer along the whole sight at the moment of light ray transition and furthermore on the sight distance and slope.

3 MATHEMATICAL MODEL OF THE VERTICAL REFRACTION

Because the determination of the meteorological parameters along the whole sight is not practicable, this led in the past to the development of several methods to be used to eliminate the vertical refraction impact [1], [7], [9], [13], [19]. Mathematical modelling comes from the idea of incorporating the measurement conditions (change of the meteorological parameters, different observational time, etc.) into a mathematical model and subsequent calculating the unknown parameters of the model with a sufficient number of the redundant measurements. Creation of a mathematical model comprises determination of the variables and equations, calculation of the model parameters, verification of a proposed model and its application.

The aim of a proposed model is to capture a course of a systematic influence among the series of elevations by means of the meteorological parameters. When defining the models we will use a k – dimensional vector of the unknown parameters $\boldsymbol{\beta}$ and m – dimensional vectors of the explanatory and interpreted variables.

3.1 Multiple regression model

The actual physical state of atmosphere at a given location determines the basic meteorological parameters. In the lowest layers is most evident the change of the air temperature and pressure. The regression model, which expresses a linear dependency of the elevation changes on the changes of the air temperature and pressure, can be written in the form:

$$h_i^{'} = \beta_0 + \beta_1 \cdot T_i^{'} + \beta_2 \cdot p_i^{'}, \tag{19}$$

for $i = 1,\ 2,\ \ldots,\ m$, where

$$h_i^{'} = h_i - \frac{1}{m} \cdot \sum_{i=1}^{m} h_i, \quad T_i^{'} = T_i - \frac{1}{m} \cdot \sum_{i=1}^{m} T_i, \quad p_i^{'} = p_i - \frac{1}{m} \cdot \sum_{i=1}^{m} p_i, \tag{20}$$

where h is a measurable elevation [m], T, p – the air temperature and pressure. The theoretical model described by the equation (19) can be expressed in a matrix form, which after the introduction of the approximate values of the unknown parameters can be written as follows:

$$\mathbf{y} = \mathbf{f}(\boldsymbol{\beta}_0) + \mathbf{A} \cdot \Delta\boldsymbol{\beta}, \tag{21}$$

where $\boldsymbol{\beta}$ is a k – dimensional vector of the unknown parameters, $\boldsymbol{\beta}_0$ – a k – dimensional vector of the approximate values of $\boldsymbol{\beta}$, $\mathbf{A} = \partial \mathbf{f}(\boldsymbol{\beta}) / \partial \boldsymbol{\beta}^{\mathrm{T}} \big|_{\beta=\beta_0}$ – an $m \times k$ – dimensional matrix of the first derivatives of a vector function $\mathbf{f}(\boldsymbol{\beta})$ with respect to $\boldsymbol{\beta}$ and quantified for the approximate values, $\Delta\boldsymbol{\beta}$ – a k – dimensional vector of the increments.

The task is to find an estimator of the unknown vector parameter $\boldsymbol{\beta}$ by means of a realization $\mathbf{w}$ of an observation vector ξ, $\mathrm{var}(\xi) = \boldsymbol{\Sigma}$. Because $\boldsymbol{\beta}$ is calculated in a form $\hat{\boldsymbol{\beta}} = \boldsymbol{\beta}_0 + \Delta\hat{\boldsymbol{\beta}}$, the subject of estimation is the vector of the increments $\Delta\boldsymbol{\beta}$ in pursuance of a realization $\mathbf{x}$ of a random vector $\boldsymbol{\eta}$, which is generated when reducing ξ by $\mathbf{f}(\boldsymbol{\beta})$, quantified for the approximate values $\boldsymbol{\beta}_0$, $\boldsymbol{\eta} = \xi - \mathbf{f}(\boldsymbol{\beta}_0)$, $\mathrm{var}(\boldsymbol{\eta}) = \boldsymbol{\Sigma}$. This task is usually solved by the least squares method, i.e. from a condition of minimizing a quadratic form [5], [11], [12]:

Volume LXI (2015), No.1
p. 32-41, ISSN 1802-5420

$$Q = \left(\mathbf{x} - \mathbf{A} \cdot \Delta\hat{\boldsymbol{\beta}}\right)^{\mathrm{T}} \cdot \boldsymbol{\Sigma}^{-1} \cdot \left(\mathbf{x} - \mathbf{A} \cdot \Delta\hat{\boldsymbol{\beta}}\right) = \min . \tag{22}$$

Minimum of the quadratic form is obtained by its derivation with respect to $\Delta\hat{\boldsymbol{\beta}}$ and by subsequent placing to equal zero. Then, the estimator $\Delta\hat{\boldsymbol{\beta}}$ of the vector $\Delta\boldsymbol{\beta}$ can be calculated according to the following:

$$\Delta\hat{\boldsymbol{\beta}} = \left(\mathbf{A}^{\mathrm{T}} \cdot \boldsymbol{\Sigma}^{-1} \cdot \mathbf{A}\right)^{-1} \cdot \mathbf{A}^{\mathrm{T}} \cdot \boldsymbol{\Sigma}^{-1} \cdot \mathbf{x} . \tag{23}$$

The estimation of the unknown parameters provides the model regularity conditions, i.e. $Rank(\mathbf{A}) = k \le m$, $Rank(\boldsymbol{\Sigma}) = m$.

The necessary part before the application of the model is its verification, which comprises an assessment of the model quality, statistical significance of the estimated parameters and analysis of the regression model assumptions. The final stage of the regression model is its application. According to the given aim, the proposed, quantified and verified model will be applied in order to reduce the impact of the vertical refraction in the following form:

$$h_i^{corr} = h_i - \left(\hat{\beta}_0 + \hat{\beta}_1 \cdot T_i^{'} + \hat{\beta}_2 \cdot p_i^{'}\right), \tag{24}$$

where h^{corr} is an elevation corrected for the vertical refraction impact.

3.2 Two-regime model

The behaviour of air pressure is related to running of the air temperature and consideration of the dependency between the explanatory variables allows extending the above mentioned regression model. We propose to use the variables dependency in order to divide a region of the air pressure in two sub-regions and in each of them will pay potentially different rules – regimes for the interpreted variable. An incorporation of this dependence into a single model requires a creation of the new explanatory variables, which are dependent on the original variables, but also among themselves. This fact then leads to a violation of the assumption about the independence of the explanatory variables and therefore when finding the suitable parameters β_j ($j = 0,\ 1,\ \ldots,\ k$) cannot be used the methods of the classic mathematical statistics. The determining equation of the two-regime model can be formulated as follows [14]:

$$h_i^{'} = \beta_0 + \beta_1 \cdot T_i^{'} + \beta_2 \cdot p_i^{'} + \beta_3 \cdot \min\left\{p_i^{'}, \hat{p}\left(T^{'}\right)_i\right\} + \beta_4 \cdot \max\left\{p_i^{'}, \hat{p}\left(T^{'}\right)_i\right\}, \tag{25}$$

or after specification into the regimes, as follows [12]:

$$h_i^{'} = \begin{cases} \beta_0 + \beta_1 \cdot T_i^{'} + (\beta_2 + \beta_3).p_i^{'} + \beta_4.p\left(T_i^{'}\right) & p_i^{'} \le p\left(T_i^{'}\right), \\ \beta_0 + \beta_1 \cdot T_i^{'} + (\beta_2 + \beta_4).p_i^{'} + \beta_3.p\left(T_i^{'}\right) & p_i^{'} > p\left(T_i^{'}\right), \end{cases} \tag{26}$$

where min and max are the aggregation functions [3], [6], which assign the smallest and biggest values from the pair of values $p_i^{'}$ and $p\left(T_i^{'}\right)$, and $p\left(T_i^{'}\right)$ is the threshold parameter defined by the threshold function. The threshold parameter splits the atmospheric pressure domain and is incorporated as an explanatory variable to increase the fitting potential of the model.

The threshold function represents an approximation of the relationship between the atmospheric pressure and temperature. The approximation of the functional relationship can be solved by different types of functions. Among the most commonly used belong polynomial functions. The polynomial of the L^{th} degree expressing the relation between the atmospheric pressure and temperature can be written as follows:

$$p\left(T^{'}\right)_i = \gamma_0 + \gamma_1 \cdot T_i^{'} + \gamma_2 \cdot \left(T_i^{'}\right)^2 + \ldots + \gamma_L \cdot \left(T_i^{'}\right)^L = \sum_{l=0}^{L} \gamma_l \cdot \left(T_i^{'}\right)^l , \tag{27}$$

for $i = 1,\ 2,\ \ldots,\ m$ and γ_L are the coefficients of the polynomial. Because the appropriate degree of the polynomial for this task is not known, we will start with the polynomial of the zero degree (constant, *L=0*), through the first degree (straight-line, *L=1*) and second degree (quadratic parabola, *L=2*) to the third degree (cubic parabola, *L=3*). The determining eq. 19 and 25 of both models can be written in a matrix form:

$$\mathbf{y} = \mathbf{f}(\boldsymbol{\beta}), \tag{28}$$

The establishment of the matrix form of the theoretical model is similar as for the regression model:

$$\mathbf{y} = \mathbf{f}\left(\boldsymbol{\beta}_0\right) + \mathbf{A} \cdot \Delta\boldsymbol{\beta} . \tag{29}$$

The difference is in the dimension extension of the vector of unknown parameters $\boldsymbol{\beta} = \boldsymbol{\beta}_0 + \Delta\boldsymbol{\beta}$ about two new parameters β_3 and β_4:

$$\boldsymbol{\beta} = (\beta_0, \quad \beta_1, \quad \beta_2, \quad \beta_3, \quad \beta_4)^{\mathrm{T}}, \tag{30}$$

which is subsequently associated with the dimension change of the vector of approximated values $\boldsymbol{\beta}_0$, unknown increments $\Delta\boldsymbol{\beta}$ and matrix $\mathbf{A} = \partial \mathbf{f}(\boldsymbol{\beta}) / \partial \boldsymbol{\beta}^{\mathrm{T}}\big|_{\beta=\beta_0}$. If the air pressure is approximated by the polynomial of the zero or first degree, then variables $T^{'}$, $p^{'}$, $\min\{p^{'}, \hat{p}(T^{'})\}$ and $\max\{p^{'}, \hat{p}(T^{'})\}$ are linearly dependent. In this case it is not necessary to consider parameter β_4 and then the vector of unknown parameters passes to the form:

$$\boldsymbol{\beta} = (\beta_0, \quad \beta_1, \quad \beta_2, \quad \beta_3)^{\mathrm{T}}. \tag{31}$$

Although the relationship of the individual explanatory variables in the mathematical model is not independent, the appropriate values of the vector parameter $\boldsymbol{\beta} = \boldsymbol{\beta}_0 + \Delta\boldsymbol{\beta}$ in the stochastic model can be determine with a classic numerical approach, i.e. with the least squares method:

$$\Delta\hat{\hat{\boldsymbol{\beta}}} = \left(\mathbf{A}^{\mathrm{T}} \cdot \boldsymbol{\Sigma}^{-1} \cdot \mathbf{A}\right)^{-1} \cdot \mathbf{A}^{\mathrm{T}} \cdot \boldsymbol{\Sigma}^{-1} \cdot \mathbf{x}, \tag{32}$$

where $\mathbf{x}$ is a realization of a random vector $\boldsymbol{\eta} = \boldsymbol{\xi} - \mathbf{f}(\boldsymbol{\beta}_0)$. The solution is conditional to the existence of all necessary inversion. These are ensured by the regularity conditions of the model, i.e. $Rank(\mathbf{A}) = k \le m$, $Rank(\boldsymbol{\Sigma}) = m$. Double roof in relation (32) means, that the calculation of the vector parameter is two-staged, i.e. into the calculation enters values, which are also the subject of the previous calculations – $\hat{p}(T^{'})_i$. An important part after the specification and quantification of the model is its verification. Because the proposed model is not built on the statistical assumptions, the verification lies only in the quality assessment. The choice of characteristic, quality assessment and comparison of the individual models with different degrees of the polynomial is a subject of next part. The final stage, i.e. the application of the model depends on the choice of the threshold function, eventually on the degree of a polynomial:

$$h_i^{corr} = h_i - \begin{cases} \hat{\hat{\beta}}_0 + \hat{\hat{\beta}}_1 \cdot T_i^{'} + \hat{\hat{\beta}}_2 \cdot p_i^{'} + \hat{\hat{\beta}}_3 \cdot \min\{p_i^{'}, \hat{p}(T^{'})_i\}, & L = 0,\ 1, \\ \hat{\hat{\beta}}_0 + \hat{\hat{\beta}}_1 \cdot T_i^{'} + \hat{\hat{\beta}}_2 \cdot p_i^{'} + \hat{\hat{\beta}}_3 \cdot \min\{p_i^{'}, \hat{p}(T^{'})_i\} + \hat{\hat{\beta}}_4 \cdot \max\{p_i^{'}, \hat{p}(T^{'})_i\}, & L = 2,\ 3, \end{cases} \tag{33}$$

where h^{corr} is an elevation corrected for the vertical refraction impact.

4 EXPERIMENTAL MEASUREMENT

The experiment has an irreplaceable place in a research, it serves to obtain or verify the theories, hypotheses and experiences. It comprises a tracking of the features, objects and their changes in a relation to the external conditions. In our case we focused on observation of the elevations changes with the changing weather conditions. We carried out together 13 experiments with length of a measurement from 11 to 14 hours per day (tab. 1). For a measurement were chosen sights with different lengths, passing over different types of surfaces and with the endpoints stabilized by means of observation pillars allowing attachment of the instruments and targets.

Tab. 1 Basis information about experiments

No.	Date	Time	Slope distance	Instrument
1	20. 11. 2007	$6^{15} – 18^{00}$ 2)	232,292 m	Trimble 3602 DR
2	20. 11. 2008	$6^{40} – 18^{30}$ 2)	183,770 m	Trimble 3602 DR
3	25. 07. 2009	$7^{00} – 20^{45}$ 2)	134,504 m	Trimble 3602 DR
4	25. 07. 2009	$7^{15} – 21^{00}$ 2)	134,504 m	Trimble 3602 DR
5	23. 07. 2010	$6^{15} – 20^{00}$ 2)	181,944 m	Trimble 3602 DR
6	26. 07. 2010	$6^{30} – 20^{20}$ 2)	181,944 m	Trimble 3602 DR
7	19. 09. 2011	$7^{00} – 19^{00}$ 1)	203,831 m	Leica TCRA 1201
8	11. 10. 2011	$7^{20} – 18^{00}$ 1)	184,547 m	Trimble 3602 DR
9	12. 03. 2012	$7^{00} – 19^{00}$ 1)	112,489 m	Trimble 3602 DR
10	31. 03. 2012	$7^{15} – 19^{15}$ 1)	318,963 m	Trimble 3602 DR
11	19. 04. 2012	$7^{00} – 19^{00}$ 1)	218,329 m	Trimble 3602 DR
12	19. 04. 2013	$7^{05} – 19^{05}$ 1)	218,329 m	Trimble VX
13	26.05.2013	$7^{00} – 19^{00}$ 1)	134,504 m	Leica TS30

1) – 1 hour lag between measurements, 2) – 2 hours lag between measurements

Each experiment was performed with the same manner, all parameters needed for further computation were measured and registered in 1 or 2 hours interval. Zenith angles were measured in 10 ranks with achieved standard deviation from 1^{cc} to $4{,}4^{cc}$. Meteorological parameters – the air temperature and pressure were measured simultaneously with the zenith angles at a station point by means of precision hydro-/thermo-/barometer equipped with a capacitive humidity sensor, resistance temperature sensor and piezoresistive pressure sensor. Height of an instrument and target was determined by measuring on a levelling staff, held at the auxiliary points near the endpoints, in two faces of a telescope (fig 3). Each elevation was also determined by means of the precise levelling method. In pursuance of the measured parameters were determined the elevations, associated to the auxiliary points A and B, for each measurement time according to the following:

$$h_{AB} = h_i + c + d_s \cdot \cos(z^m) - h_t , \tag{34}$$

where h_i and h_t is the height of an instrument and target, c – the Earth curvature correction ($c = s^2 / 2 \cdot R$), s – the horizontal distance, R – the radius of the Earth, d_s – the slope distance, z_{IJ}^m – the measured zenith angle (fig. 3). In calculation was not taken into account the influence of a deflection of the vertical and the vertical refraction impact.

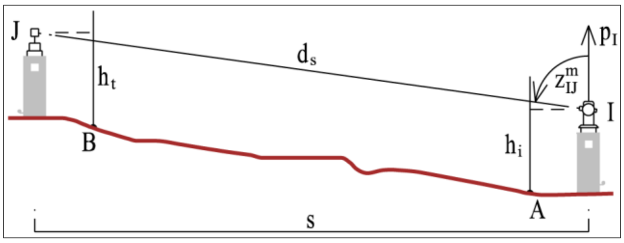

Fig. 3 Determination of an elevation

5 EVALUATION OF THE RESULTS

To assess the quality of the proposed models we chose as a criterion the residual sum of squares. The residual sum of squares expresses the part of variability of the interpreted variable, which is not explained by the model and therefore a lower value of this criterion indicates a better built model, which more closely reflects the modelled data. The comparison of the residual sums of squares between the regression model and two-regime model with the threshold function of the different degrees of the polynomial is given in tab. 2. For convenience, we also calculated relative percentage improvements of the two-regime model due to the regression model:

$$I = \frac{SS_{RM} - SS_{TM}}{SS_{RM}} \cdot 100 , \tag{35}$$

where I is the relative improvement in [%], SS_{RM} and SS_{TM} is the residual sum of squares from the regression model and two-regime model (tab. 3).

Tab. 2 Quality comparison of the proposed models

No.	Regression model	Two-regime model (L – degree of polynomial)			
		$L = 0$	$L = 1$	$L = 2$	$L = 3$
	[mm^2]	[mm^2]	[mm^2]	[mm^2]	[mm^2]
1	0,62	0,53	0,53	**0,14**	**0,14**
2	0,83	**0,37**	0,75	0,64	0,60
3	0,96	0,79	0,71	**0,67**	0,81
4	0,71	0,62	0,57	**0,50**	0,63
5	3,24	1,29	1,07	**0,13**	0,62
6	4,09	1,82	1,97	1,38	**0,45**
7	6,75	5,82	4,67	**1,96**	4,16
8	3,50	3,31	3,47	**1,30**	1,52
9	1,24	1,15	1,24	**0,61**	0,71
10	10,60	10,32	8,53	**3,41**	7,37
11	6,32	3,90	4,44	**1,82**	2,35
12	6,06	3,60	3,85	**1,36**	2,43
13	6,40	3,57	3,80	3,13	**2,90**

Tab. 3 Relative percentage improvements

No.	Two-regime model (L – degree of polynomial)			
	$L=0$	$L=1$	$L=2$	$L=3$
	[%]	[%]	[%]	[%]
1	14,5	14,5	**77,4**	**77,4**
2	**55,4**	9,6	22,9	27,7
3	17,7	26,0	**30,2**	15,6
4	12,7	19,7	**29,6**	11,3
5	60,2	67,0	**96,0**	80,9
6	55,5	51,8	66,3	**89,0**
7	13,8	30,8	**71,0**	38,4
8	5,4	0,9	**62,9**	56,6
9	7,3	0,0	**50,8**	42,7
10	2,6	19,5	**67,8**	30,5
11	38,3	29,7	**71,2**	62,8
12	40,6	36,5	**77,6**	59,9
13	41,5	37,7	48,7	**52,5**

The above mentioned comparison of the residual sums of squares and the relative percentage improvements documents an increase of the quality of the two-regime model compared to the regression model. Except the experiments no. 2, 6 and 13, the two-regime model with the quadratic threshold function achieves the highest improvements. Based on the selected criterion to assess the quality, we can mark this model as the most optimal from all tested and in the next part can be used in order to reduce the impact of the vertical refraction. Practical use of the chosen model is graphically presented in fig. 4 and 5, which show time course of measured elevations (h), elevations acquired by the model (h^{corr}) and elevation determined be means of the precise levelling (h_{PL}).

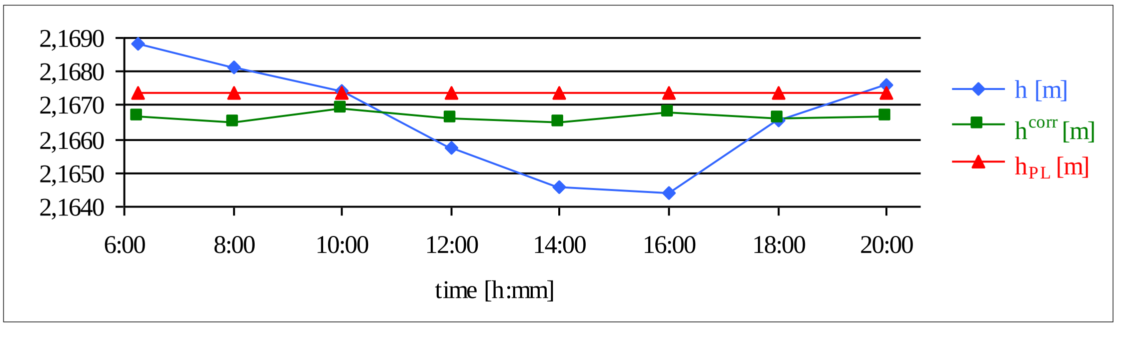

Fig. 4 Graphical comparison of elevations (experiment no. 5)

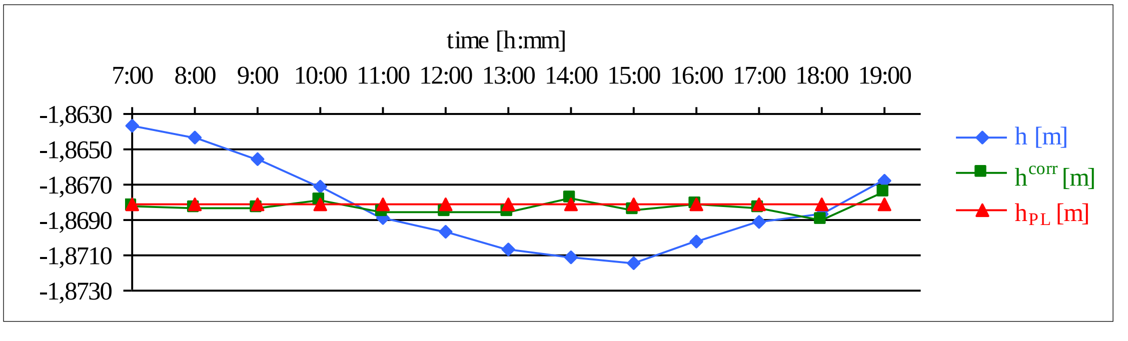

Fig. 5 Graphical comparison of elevations (experiment no. 11)

Volume LXI (2015), No.1
p. 32-41, ISSN 1802-5420

6 CONCLUSION

Changeable environment, where the measurements are realized, effects on the results of these measurements and therefore an introduction of the corrections and the elimination of its influence represent one of the possibilities of increasing the reliability and accuracy of the acquired results. The trigonometric measurement of elevations is a well-known surveying technique which is mainly influenced by the vertical refraction. In order to eliminate the impact of the vertical refraction from the results of repeated measurements we have established a mathematical model. Besides the classical regression model, we have suggested the two-regime with the threshold function showing the relationship of the air pressure and temperature. Switching between two regimes provides the aggregation functions min and max. To calculate the unknown parameters, we used the least squares method. The contribution of the two-regime model versus the regression model is documented by the comparison of the residual sums of squares. Given comparison and the achieved relative improvements have shown that the two-regime model contributes in higher rate to the variability explanation of the modelled elevations and that in any case doesn't provide lower quality than the regression model. On the other hand the disadvantage of the model is the need of calculation of the input variables and also more unknown parameters, what according to the model regularity conditions yield to increasing the number of measurements. According to the comparison of the residual sums of squares we chose model with the quadratic threshold function as the most suitable model from the proposed models. The comparison of match between the corrected elevations and the elevations determined by precise levelling has shown that the consideration of the meteorological parameters significantly contributes to the elimination of vertical refraction but doesn't lead to the complete exclusion of the vertical refraction. This fact is on one side determined by the quality of the used model and on the other side it is a consequence of fluctuations in the systematic influence, which take a variable character and may not converge to the zero mean value.

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RESUMÉ

Na charakterizovanie priestorovej polohy bodov pomocou terestrických geodetických metód potrebujeme okrem polohového merania (smery a dĺžky) vykonať aj výškové meranie (prevýšenia). Podľa prístrojového vybavenia a rôznych princípov určovania prevýšení vznikol v geodézii celý rad metód výškových meraní. Každá metóda je charakteristická použitým prístrojom a pomôckami, postupom merania, dosahovanou presnosťou výsledkov merania a efektívnosťou využitia pri danom účele a podmienkach.

Trigonometrické meranie prevýšení je bežnou metódou v geodetickej praxi. Napriek nespornému pokroku a neustálej modernizácii meracích prístrojov, hlavnou prekážkou použitia trigonometrickej metódy sú zmeny meteorologických prvkov, ktoré najvýraznejšie ovplyvňujú presnosť merania zenitových uhlov. Jednou z možností zvyšovania spoľahlivosti a presnosti dosiahnutých výsledkov predstavuje výpočet korekcií prostredníctvom matematicko-štatistických modelov.